Problem Set 25: L'Hôpital's Rule

Key skills: Derivatives, Limits

Practice Problems

Applying L'Hôpital's rule

Use L'Hôpital's rule to calculate the following limits.

$a)\ \lim_{x\to 0} \frac{\sin x}{x}$ $\quad b)\ \lim_{x\to 0} \frac{x}{\sin x}$ $\quad c)\ \lim_{x\to 0} \frac{\tan x}{x}$ $\quad d)\ \lim_{x\to 1} \frac{x-1}{x^2-1}$

$e)\ \lim_{x\to \infty} \frac{\ln x}{x}$ $\quad f)\ \lim_{x\to \infty} \frac{x+e^x}{xe^x}$ $\quad g)\ \lim_{x\to \infty} \frac{\frac{1}{x^4}-\frac{1}{x^2}}{\ln x}$ $\quad h)\ \lim_{x\to \infty} \frac{\ln(x^2)}{(\ln x)^2}$

Forbidden questions

Explain why you *cannot* apply L'Hôpital's rule for any of the following limits.

$a)\ \lim_{x\to \pi} \frac{\sin x}{x}$ $\quad b)\ \lim_{x\to 0} \frac{\cos x}{x}$ $\quad c)\ \lim_{x\to -1} \frac{x-1}{x^2-1}$

$d)\ \lim_{x\to 4} \frac{x^2-16}{e^x}$ $\quad e)\ \lim_{x\to 4} \frac{x^2}{e^x}$ $\quad f)\ \lim_{x\to \infty} \frac{e^{-x}}{x}$

Answers

Answers to: Applying L'Hôpital's rule

$$a)\quad \lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\cos x}{1}=1$$

$$b)\quad \lim_{x\to 0}\frac{x}{\sin x}=\lim_{x\to 0}\frac{1}{\cos x}=1$$

$$c)\quad \lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{\sec^2 x}{1}=1$$

$$d)\quad \lim_{x\to 1}\frac{x-1}{x^2-1}=\lim_{x\to 1}\frac{1}{2x}=\frac{1}{2}$$

$$e)\quad \lim_{x\to \infty}\frac{\ln x}{x}=\lim_{x\to \infty}\frac{\frac{1}{x}}{1}=\lim_{x\to \infty}\frac{1}{x}=0$$

$$f)\quad \lim_{x\to \infty}\frac{1+e^x}{xe^x}=\lim_{x\to \infty}\frac{e^x}{xe^x+e^x}=\lim_{x\to \infty}\frac{1}{x+1}=0$$

$$g)\quad \lim_{x\to \infty}\frac{\frac{1}{x^4}-\frac{1}{x^2}}{\ln x}=\lim_{x\to \infty}\frac{-\frac{4}{x^5}+\frac{2}{x^3}}{\frac{1}{x}}=\lim_{x\to \infty}\frac{-\frac{4}{x^4}+\frac{2}{x^2}}{=}0$$

$$h)\quad \lim_{x\to \infty}\frac{\ln(x^2)}{(\ln x)^2}=\lim_{x\to \infty}\frac{\frac{1}{x^2}2x}{2(\ln x)\frac{1}{x}}=\lim_{x\to \infty}\frac{1}{\ln x}=0$$

Answers to: Forbidden questions

L'Hôpital's rule only applies to limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$. This does not describe any of the limits in this section. Specifically:

$$a)\quad \lim_{x\to \pi}\frac{\sin x}{x}=\frac{0}{1}=0$$

$$b)\quad \lim_{x\to 0}\frac{\cos x}{x}\text{ has the form }\frac{1}{0}$$

$$c)\quad \lim_{x\to -1}\frac{x-1}{x^2-1}\text{ has the form }\frac{-2}{0}$$

$$d)\quad \lim_{x\to 4}\frac{x^2-16}{e^x}=\frac{0}{e^{16}}=0$$

$$e)\quad \lim_{x\to 4}\frac{x^2}{e^x}=\frac{16}{e^{16}}$$

$$f)\quad \lim_{x\to \infty}\frac{e^{-x}}{x}\text{ has the form }\frac{0}{\infty}$$