

## Problem Set 23: Integration using Substitution

Key skills: Substitution

### Practice Problems

Do the following integrals by substitution. Clearly label your chosen substitution.

$$\begin{array}{lll} a) \int \frac{2x^2}{\sqrt{1-4x^3}} dx & b) \int 2x(x^2-2)^{80} dx & c) \int x^7(\cos x^8) dx \\ d) \int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}} dx & e) \int (x^3+x)^7(3x^2+1) dx & f) \int \cos x \sin^4 x dx \\ g) \int_0^4 \frac{4x}{\sqrt{4+x^2}} dx & h) \int_0^{\pi/2} \cos x \sin^2 x dx & i) \int_0^3 \frac{x^2+1}{\sqrt{x^3+3x+4}} dx \\ j) \int_1^{e^2} \frac{\ln x}{x} dx & & \end{array}$$

**Answers**

$$a) \int \frac{2x}{\sqrt{1-4x^3}} dx = \frac{-\sqrt{1-4x^3}}{3} \quad u = 1 - 4x^3 \quad du = -12x^2 dx$$

$$b) \int 2x(x^2 - 2)^{80} dx = \frac{(x^2 - 2)^{81}}{81} \quad u = x^2 - 2 \quad du = 2x dx$$

$$c) \int x^7(\cos x^8) dx = \frac{-\sin x^8}{8} \quad u = x^8 \quad du = 8x^7 dx$$

$$d) \int \frac{(\sqrt{x} + 1)^4}{2\sqrt{x}} dx = \frac{(\sqrt{x} + 1)^5}{5} \quad u = \sqrt{x} + 1 \quad du = \frac{1}{2\sqrt{x}} dx$$

$$e) \int (x^3 + x)^7(3x^2 + 1) dx = \frac{(x^3 + x)^8}{8} \quad u = x^3 + x \quad du = (3x^2 + 1) dx$$

$$f) \int \cos x \sin^4 x dx = \frac{\sin^5 x}{5} \quad u = \sin x \quad du = \cos x dx$$

$$g) \int_0^4 \frac{4x}{\sqrt{4+x^2}} dx = \frac{32}{2}(5\sqrt{5} - 1) \quad u = 4 + x^2 \quad du = 2x dx$$

$$h) \int_0^{\pi/2} \cos x \sin^2 x dx = \frac{1}{3} \quad u = \sin x \quad du = \cos x dx$$

$$i) \int_0^3 \frac{x^2 + 1}{\sqrt{x^3 + 3x + 4}} dx = \frac{2\sqrt{40} - 4}{3} \quad u = x^3 + 3x + 4 \quad du = (3x^2 + 3) dx$$

$$j) \int_1^{e^2} \frac{\ln x}{x} dx = 2 \quad u = \ln x \quad du = \frac{1}{x} dx$$