

Problem Set 19: Linear Approximation

Key skills: Tangent Lines as approximations of functions

Practice Problems

Write the linear approximation (tangent line) of the given function at the given points. Use it to calculate nearby values. If you have a computer to give approximate values, compare the results from your approximation to what your computer gives.

- (a) The function $f(x) = \sin x$ near the points $(0, 0)$ and $(\pi, 0)$.
- (b) The function $f(x) = \cos x$ near the point $(0, 1)$ and $(2\pi, 1)$.
- (c) The function $f(x) = \tan x$ near the point $(0, 0)$.
- (d) The function $f(x) = x^2$ near the points $(0, 0)$ and $(1, 1)$.
- (e) The function $f(x) = x^3$ near the points $(0, 0)$ and $(1, 1)$.
- (f) The function $f(x) = (x - 5)^2$ near the points $(5, 0)$ and $(0, 25)$.
- (g) The function $f(x) = e^x$ near the point $(0, 1)$.
- (h) The function $f(x) = \frac{x+1}{2x^2-1}$ near the points $(0, -1)$ and $(1, 2)$.

Answers

- (a) $\frac{d}{dx} \sin x = \cos x$
Near $(0, 0)$: $f(x) \approx x$
Near $(\pi, 0)$: $f(x) \approx -x + \pi$
- (b) $\frac{d}{dx} \cos x = -\sin x$
Near $(0, 1)$: $f(x) \approx 1$
Near $(2\pi, 1)$: $f(x) \approx 1$
- (c) $\frac{d}{dx} \tan x = \sec^2 x$
Near $(0, 0)$: $f(x) \approx x$
- (d) $\frac{d}{dx} x^2 = 2x$
Near $(0, 0)$: $f(x) \approx 0$
Near $(1, 1)$: $f(x) \approx 2x - 1$
- (e) $\frac{d}{dx} x^3 = 3x^2$
Near $(0, 0)$: $f(x) \approx 0$
Near $(1, 1)$: $f(x) \approx 3x - 2$
- (f) $\frac{d}{dx} (x - 5)^2 = 2x - 10$
Near $(5, 0)$: $f(x) \approx 0$
Near $(0, 25)$: $f(x) \approx -10x + 25$
- (g) $\frac{d}{dx} e^x = e^x$
Near $(0, 1)$: $f(x) \approx x + 1$
- (h) $\frac{d}{dx} \frac{x+1}{2x^2-1} = -\frac{2x^2+4x+1}{(2x^2-1)^2}$
Near $(0, -1)$: $f(x) \approx -x - 1$
Near $(1, 2)$: $f(x) \approx -7x + 9$