Problem Set 18: Higher Derivatives

Key skills: Higher Derivatives (a.k.a. Multiple Derivatives)

Practice Problems

Watch for patterns in your solutions to some of these problems.

The patterns you find in problems c, g, h, i, and j are particularly interesting and useful.

$a)\frac{d^2}{dx^2}x^4 - 3x + 1$ $b)\frac{d^2}{dx^2}\sin(x^2)$ $c)\frac{d^2}{dx^2}e^x$

$d)\frac{d^2}{dx^2}\cos(x^2 - 1)$ $e)\frac{d^2}{dx^2}\frac{x+1}{x^2+1}$ $f)\frac{d^2}{dx^2}\ln(1 - x^2)$

$g)\frac{d^3}{dx^3}x^4 - 5x^3 + 6x + 3$ $h)\frac{d^5}{dx^5}x^4 - 5x^3 + 6x + 3$

$i)\frac{d^4}{dx^4}\sin(4x)$ $j)\frac{d^4}{dx^4}e^{6x+1}$ $k)\frac{d^3}{dx^3}e^{2x}$

Answers

$$a) \frac{d^2}{dx^2} x^4 - 3x + 1 = \frac{d}{dx} 4x^3 - 3 = 12x^2$$

$$b) \frac{d^2}{dx^2} \sin(x^2) = \frac{d}{dx} 2x\cos(x^2) = 2\cos(x^2) - 4x^2\sin(x^2)$$

$$c) \frac{d^2}{dx^2} e^x = \frac{d}{dx} e^x = e^x$$

$$d) \frac{d^2}{dx^2} \cos(x^2 - 1) = \frac{d}{dx} - 2x\sin(x^2 - 1) = -2\sin(x^2 - 1) - 4x^2\cos(x^2 - 1)$$

$$\begin{aligned} e) \frac{d^2}{dx^2} \frac{x+1}{x^2+1} &= \frac{d}{dx} \frac{x^2 + 1 - (x+1)(2x)}{(x^2+1)^2} = \frac{d}{dx} \frac{-x^2 - 2x + 1}{(x^2+1)^2} \\ &= \frac{(-2x-2)(x^2+1)^2 - (-x^2 - 2x + 1)2(x^2+1)(2x)}{(x^2+1)^4} \\ &== \frac{(-2x-2)(x^2+1) - (-x^2 - 2x + 1)(4x)}{(x^2+1)^3} \\ &= \frac{-2x^3 - 2x^2 - 2x - 2 - (-4x^3 - 8x^2 + 4x)}{(x^2+1)^3} = \frac{2x^3 + 6x^2 - 6x - 2}{(x^2+1)^3} \end{aligned}$$

$$f) \frac{d^2}{dx^2} \ln(1 - x^2) = \frac{d}{dx} \frac{-2x}{1 - x^2} = \frac{-2(1 - x^2) - (-2x)(-2x)}{(1 - x^2)^2} = \frac{-2x^2 - 2}{(1 - x^2)^2}$$

$$g) \frac{d^3}{dx^3} x^4 - 5x^3 + 6x + 3 = \frac{d^2}{dx^2} 4x^3 - 15x^2 + 6 = \frac{d}{dx} 12x^2 + 30x = 24x + 30$$

$$h) \frac{d^5}{dx^5} x^4 - 5x^3 + 6x + 3 = \frac{d^4}{dx^4} 4x^3 - 15x^2 + 6 = \frac{d^3}{dx^3} 12x^2 + 30x = \frac{d^2}{dx^2} 24x + 30 = \frac{d}{dx} 24 = 0$$

$$i)\frac{d^4}{dx^4}\sin(4x) = \frac{d^3}{dx^3}4\cos(4x) = \frac{d^2}{dx^2}(-16\sin(4x)) = \frac{d}{dx}(-64\cos(4x)) = 256\sin(4x)$$

$$j)\frac{d^4}{dx^4}e^{6x+1} = \frac{d^3}{dx^3}6e^{6x+1} = \frac{d^2}{dx^2}36e^{6x+1} = \frac{d}{dx}216e^{6x+1} = 1296e^{6x+1}$$

or $$\frac{d^4}{dx^4}e^{6x+1} = \frac{d^3}{dx^3}6e^{6x+1} = \frac{d^2}{dx^2}6^2e^{6x+1} = \frac{d}{dx}6^3e^{6x+1} = 6^4e^{6x+1}$$

$$k)\frac{d^3}{dx^3}e^{2x} = \frac{d^2}{dx^2}2e^{2x} = \frac{d}{dx}4e^{2x} = 8e^{2x}$$