Problem Set 17: Implicit Derviatives

Key skills: Implicit Differentiation, Chain Rule, Tangent Lines

Practice Problems

For each of the following curves, find a general expression for the implicit derivative $\frac{dy}{dx}$, determine where the derivative is undefined, and calculate the tangent line at the given points.

It is strongly recommended that you first plot each curve, using Desmos or another curve-graphing program.

a) $x^2 + y^2 = 36$, tangent at $(0, 6)$, at $(3\sqrt{2}, 3\sqrt{2})$, and at $(-3\sqrt{2}, 3\sqrt{2})$.

b) $x^2 - y^2 = 36$, tangent at $(\sqrt{1000036}, 1000)$

c) $y^2 - x = 0$, tangent at $(1, 1)$, and at $(1, -1)$

d) $x^2 - y = 0$, tangent at $(1, 1)$

e) $y^2 + \cos x = 1$, tangent at $(\pi, \sqrt{2})$, and at $\left(\frac{\pi}{2}, -1\right)$

Answers

a) $\dfrac{dy}{dx} = -\dfrac{x}{y}$

$(0,6)$: The slope is 0, so the tangent line is $y = 6$ (a constant).

$(3\sqrt{2},\, 3\sqrt{2})$: The slope is -1. To find the intercept: $(3\sqrt{2}) = -(3\sqrt{2}) + b$, so $b = 6\sqrt{2}$. Then the tangent line is $y = -x + 6\sqrt{2}$.

$(-3\sqrt{2},\, 3\sqrt{2})$: The slope is 1. To find the intercept: $(3\sqrt{2}) = (-3\sqrt{2}) + b$, so again $b = 6\sqrt{2}$. (You can see this from symmetry if you sketch these tangent lines on the curve.) Then the tangent line is $y = x + 6\sqrt{2}$

b) $\dfrac{dy}{dx} = \dfrac{x}{y}$

$(\sqrt{1000036},\, 1000)$: The slope is $\frac{\sqrt{1000036}}{1000} \approx 1.000018 \approx 1$. To a very good approximation, the tangent line is $y = x$.

c) $\dfrac{dy}{dx} = \dfrac{1}{2y}$

$(1,1)$: The slope is $1/2$. To find the intercept: $(1) = \frac{1}{2}(1) + b$, so $b = 1/2$. Then the tangent line is $y = \frac{1}{2}x + \frac{1}{2}$.

$(1,-1)$: The slope is $-1/2$. To find the intercept: $(-1) = -\frac{1}{2}(1) + b$, so $b = -1/2$. Then the tangent line is $y = -\frac{1}{2}x - \frac{1}{2}$.

d) $\dfrac{dy}{dx} = 2x$

$(1,1)$: The slope is 2. To find the intercept: $(1) = 2(1) + b$, so $b = -1$. Then the tangent line is $y = 2x - 1$.

e) $\dfrac{dy}{dx} = \dfrac{\sin x}{2y}$

$(\pi, \sqrt{2})$: The slope is 0, so the tangent line is $y = \sqrt{2}$ (a constant).

$\left(\frac{\pi}{2}, -1\right)$: The slope is $-\frac{1}{2}$. To find the intercept: $-1 = -\frac{1}{2}\left(\frac{\pi}{2}\right) + b$, so $b = \frac{\pi}{4} - 1$. Then the tangent line is $y = -\frac{1}{2}x + \frac{\pi}{4} - 1$.