Problem Set 16: Derivatives

Key skills: Derivatives using Derivative Rules; Derivatives of inverse functions

Combines material from Lectures 14, 15, and 16.

Practice Problems

Derivative Rules

Do the following derivatives, explicitly labelling the rules you are using. (Power, Linearity, Product, Quotient, Chain). Simplify if you want, and if you see an obvious simplification, but don't worry about simplifying difficult expressions.

The derivative $\frac{d}{dx}e^{kx}$ (problem i) is particularly useful, and it's worth memorizing the answer once you find it. There are (at least) two different ways to do problem b.

$a)\quad \dfrac{d}{dx}x^2 \sin x \ln x \qquad b)\quad \dfrac{d}{dx}\dfrac{a^x}{b^x} \qquad c)\quad \dfrac{d}{dx}\cos(x^2+1) \qquad d)\quad \dfrac{d}{dx}\sin(\cos(\sin x)))$

$e)\quad \dfrac{d}{dx}\sqrt{1-\dfrac{1}{x^2+1}} \qquad f)\quad \dfrac{d}{dx}x^2e^x + x^3e^{x-1} \qquad g)\quad \dfrac{d}{dx}e^{\sin x+\cos x} \qquad h)\quad \dfrac{d}{dx}2^{x^4}$

$i)\quad \dfrac{d}{dx}e^{kx} \qquad j)\quad \dfrac{d}{dx}2^{x^2+x}+x^2$

Derivatives of Inverse Functions

Do the following derivatives using the rule for derivatives of inverse functions.

$k)\quad \dfrac{d}{dx}\sqrt{x} \qquad l)\quad \dfrac{d}{dx}\sqrt{x^2-1} \qquad m)\quad \dfrac{d}{dx}\ln(x^2)$

Answers

Derivative Rules

$a)$ $\frac{d}{dx}x^2 \sin x \ln x = x^2 \cos x \ln x + 2x \sin x \ln x + x \sin x$ $\qquad b)$ $\frac{d}{dx}\frac{a^x}{b^x} = \frac{a^x(\ln a - \ln b)}{b^x}$

$c)$ $\frac{d}{dx}\cos(x^2+1) = -2x\sin(x^2+1)$ $\qquad d)$ $\frac{d}{dx}\sin(\cos(\sin x)) = -\cos(\cos(\sin x)))\sin(\sin x)\cos x$

$e)$ $\frac{d}{dx}\sqrt{1-\frac{1}{x^2+1}} = \frac{1}{2\sqrt{1-\frac{1}{x^2+1}}}\frac{2x}{(x^2+1)^2}$ $\qquad f)$ $\frac{d}{dx}x^2e^x+x^3e^{x-1} = 2xe^x+x^2e^x+3x^2e^{x-1}+x^3e^{x-1}$

$g)$ $\frac{d}{dx}e^{\sin x+\cos x} = (\cos x - \sin x)e^{\sin x+\cos x}$ $\qquad h)$ $\frac{d}{dx}2^{x^4} = 2^{x^4}4x^3\ln 2$

$i)$ $\frac{d}{dx}e^{kx} = ke^{kx}$ $\qquad j)$ $\frac{d}{dx}2^{x^2+x}+x^2 = 2^{x^2+x}\ln 2(2x+2)+2x$

Derivatives of Inverse Functions

$k)$ $\frac{d}{dx}\sqrt{x}$

Use $f(x) = x^2$ so $f^{-1}(x) = \sqrt{x}$.

$$\frac{d}{dx}\sqrt{x} = \left.\frac{1}{2u}\right|_{u=\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$l)$ $\frac{d}{dx}\sqrt{x^2-1}$

Use $f(x) = (x+1)^2$ so $f^{-1}(x) = \sqrt{x^2-1}$

$$\frac{d}{dx}\sqrt{x^2-1} = \left.\frac{1}{2(u+1)}\right|_{u=\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

$m)$ $\frac{d}{dx}\ln(x^2)$

Use $f(x) = \sqrt{e^x}$ so $f^{-1}(x) = \ln x^2$

$$\frac{d}{dx}\ln(x^2) = \left.\frac{1}{\frac{e^u}{2\sqrt{e^u}}}\right|_{u=\ln x^2} = \frac{2\sqrt{e^{\ln x^2}}}{e^{\ln x^2}} = \frac{2x}{x^2} = \frac{2}{x}$$

(This last can be simplified a great deal by writing $\ln(x^2) = 2\ln x$, but it useful to do in this way to see how to use the inverse function derivative process.)