Problem Set 13: Continuity

Key skills: Continuity, Piecewise Functions

Practice Problems

For each of these functions, determine whether or not the function is continuous at the given x value.

$a)\ f(x) = x^2 \quad \text{at } x = 0 \qquad b)\ f(x) = \dfrac{1}{x^2} \quad \text{at } x = 0 \qquad c)\ f(x) = \dfrac{x+2}{(x+2)(x+4)} \quad \text{at } x = -2$

For each of these piecewise functions, determine whether or not the function is continuous at its crossover point.

$$d)\ f(x) = \begin{cases} 2 & x \le 1 \\ 4 & x > 1 \end{cases} \qquad e)\ f(x) = \begin{cases} 2x & x < 1 \\ x^2 + 3x & x \ge 1 \end{cases} \qquad f)\ f(x) = \begin{cases} 3x+2 & x < 2 \\ x^3 & x = 2 \\ 4x^2 - 8 & x > 2 \end{cases}$$

For each of these piecewise functions, find a value for a which makes the function continuous, or show that no such value exists.

$$g)\ f(x) = \begin{cases} x^2 + 3x - 1 & x \ge 2 \\ \frac{a}{x} & x < 2 \end{cases} \qquad h)\ f(x) = \begin{cases} x^2 + 3x - 1 & x \ge 2 \\ \frac{a}{x-2} & x < 2 \end{cases}$$

Answers

a) $\lim_{x\to 0^-} f(x) = 0$. $\lim_{x\to 0^+} f(x) = 0$. $f(0) = 0$. Continuous.

b) Discontinuous; $f(0)$ is not defined.

c) Discontinuous; $f(-2)$ is not defined. (Note that the $(x+2)$ factors cancel, but they can *only* be cancelled where $x + 2 \neq 0$.)

d) Discontinuous; $f(x)$ approaches (horizontally!) different values from each direction.

e) $\lim_{x\to 1^-} 2x = 2$. $\lim_{x\to 1^+} x^2 + 3x = 4$. Discontinuous.

f) $\lim_{x\to 2^-} 3x + 2 = 8$. $(2)^3 = 8$. $\lim_{x\to 2^+} 4x^2 - 8 = 8$. Continuous.

g) $\lim_{x\to 2^+} f(x) = 9$. $\lim_{x\to 2^-} f(x) = \frac{a}{2}$. Continuous if $a = 18$.

h) $\lim_{x\to 2^+} f(x) = 9$. $\lim_{x\to 2^-} f(x) = -\infty$. Not continuous for any a.