Problem Set 12: Limits, Asymptotics, and Models

Key skills: Asymptotic analysis; Mathematical models

Practice Problems

For the following population models, determine the initial population value p at $t = 0$ and the long term behaviour of the population. If the population model has a long-term steady state, determine the final population value. Classify the growth: which are exponential, which are logistic, and which are some other type of growth?

Assume that t is measured in years and p is measured in thousands of individuals.

a) $p(t) = 100e^{\frac{1}{2}t}$

b) $p(t) = 20e^{-6t}$

c) $p(t) = \dfrac{1200e^{\frac{5}{9}t}}{88 + 12\left(e^{\frac{5}{9}t}\right)}$

d) $p(t) = \dfrac{50e^{\frac{1}{10}t}}{23 + 2\left(e^{\frac{1}{10}t}\right)}$

e) $p(t) = \dfrac{50e^{\frac{1}{10}t}}{23\cos(6\pi t) + 2\left(e^{\frac{1}{10}t}\right)}$

f) $p(t) = \dfrac{200\left(\frac{t}{3}\right)^2}{35 + 5\left(\frac{t}{3}\right)^2}$

g) $p(t) = 90\left(\dfrac{t}{7}\right)^2 + 10$

h) $p(t) = \dfrac{300}{\frac{t}{2} + 1}$

i) $p(t) = \dfrac{200\left(\frac{t}{6}\right)}{77e^{\frac{1}{6}t}}$

j) $p(t) = \dfrac{9000t^2 + 900t + 9}{450t^2 - 31t + 3}$

k) $p(t) = \dfrac{22t^3 + 396t^2 - 88t + 8}{511t^2 + 2t + 4}$

l) $p(t) = \dfrac{64t^2 + 128t}{27t^3 + 9t + 3}$

Answers

a) $p(0) = 100$. The population grows without limit. This is an exponential growth model.

b) $p(0) = 20$. The population eventually decreases to 0. This is an exponential decay model.

c) $p(0) = \frac{1200}{88+12} = 12$. The population eventually stabilizes at $\frac{1200}{12} = 100$. This is a logistic growth model.

d) $p(0) = \frac{50}{23+2} = 2$. The population eventually stabilizes at $\frac{50}{2} = 25$. This is a logistic growth model.

e) $p(0) = \frac{50}{23+2} = 2$. The population eventually stabilizes at $\frac{50}{2} = 25$. (As t grows the cosine term becomes insignificant asymptotically.) This is not any of the types of growth models listed.

f) $p(0) = 0$. The population eventually stabilizes at $\frac{200}{5} = 40$. This is not any of the types of growth models listed.

g) $p(0) = 10$. The population grows without limit. This is not any of the types of growth models listed.

h) $p(0) = 300$. The population eventually decreases to 0. This is not any of the types of growth models listed.

i) $p(0) = 0$. The population eventually decreases to 0. This is not any of the types of growth models listed.

j) $p(0) = 3$. The population eventually stabilizes at $\frac{9000}{450} = 20$. This is not any of the types of growth models listed.

k) $p(0) = 2$. The population grows without limit. This is not any of the types of growth models listed.

l) $p(0) = 0$. The population becomes positive and eventually decreases back to 0. This is not any of the types of growth models listed.