Problem Set 11: Limits at Infinity

Includes practice problems for Lectures 10 and 11.

Key skills: Horizontal asymptotes; Asymptotic analysis

See last page for more advanced practice problems, with answers.

Practice Problems

a) $\lim_{x\to\infty} \dfrac{\sin x}{x^2 \cos x}$ *b)* $\lim_{x\to-\infty} \dfrac{\sin x}{x^2 \cos x}$

c) $\lim_{x\to\infty} \dfrac{\cos 3x}{x^2+2}$ *d)* $\lim_{x\to-\infty} \dfrac{\cos 3x}{x^2+2}$

e) $\lim_{x\to\infty} \dfrac{3x^3+2x+10}{9x^3-4x^2-100}$ *f)* $\lim_{x\to-\infty} \dfrac{3x^3+2x+10}{9x^3-4x^2-100}$

g) $\lim_{x\to\infty} \dfrac{5x^5-27x+2}{20x^4+20x+7}$ *h)* $\lim_{x\to-\infty} \dfrac{5x^5-27x+2}{20x^4+20x+7}$

i) $\lim_{x\to\infty} \dfrac{1}{x} - \dfrac{1}{x^2}$ *j)* $\lim_{x\to-\infty} \dfrac{1}{x} - \dfrac{1}{x^2}$

k) $\lim_{x\to\infty} \dfrac{e^{2x}x^3}{e^x x^6}$ *l)* $\lim_{x\to-\infty} \dfrac{e^{2x}x^3}{e^x x^6}$

m) $\lim_{x\to\infty} \dfrac{e^{x^2}x^3}{e^x x^6}$ *n)* $\lim_{x\to-\infty} \dfrac{e^{x^2}x^3}{e^x x^6}$

o) $\lim_{x\to\infty} \dfrac{(\ln|x|)^6}{x}$ *p)* $\lim_{x\to-\infty} \dfrac{(\ln|x|)^6}{x}$

q) $\lim_{x\to\infty} \dfrac{\sin^2 x}{x} - \dfrac{\cos^2 x}{x}$ *r)* $\lim_{x\to-\infty} \dfrac{\sin^2 x}{x} - \dfrac{\cos^2 x}{x}$

s) $\lim_{x\to\infty} \dfrac{\sqrt[3]{x^9+\sin^2 x}}{x^4+12}$ *t)* $\lim_{x\to-\infty} \dfrac{\sqrt[3]{x^9+\sin^2 x}}{x^4+12}$

Answers

$a)\ \lim_{x\to\infty} \dfrac{\sin x}{x^2\cos x}$ DNE $\qquad b)\ \lim_{x\to-\infty} \dfrac{\sin x}{x^2\cos x}$ DNE

$c)\ \lim_{x\to\infty} \dfrac{\cos 3x}{x^2+2} = 0 \qquad d)\ \lim_{x\to-\infty} \dfrac{\cos 3x}{x^2+2} = 0$

$e)\ \lim_{x\to\infty} \dfrac{3x^3+2x+10}{9x^3-4x^2-100} = \dfrac{1}{3} \qquad f)\ \lim_{x\to-\infty} \dfrac{3x^3+2x+10}{9x^3-4x^2-100} = \dfrac{1}{3}$

$g)\ \lim_{x\to\infty} \dfrac{5x^5-27x+2}{20x^4+20x+7} = \infty \qquad h)\ \lim_{x\to-\infty} \dfrac{5x^5-27x+2}{20x^4+20x+7} = -\infty$

$i)\ \lim_{x\to\infty} \dfrac{1}{x} - \dfrac{1}{x^2} = 0 \qquad j)\ \lim_{x\to-\infty} \dfrac{1}{x} - \dfrac{1}{x^2} = 0$

$k)\ \lim_{x\to\infty} \dfrac{e^{2x}x^3}{e^x x^6} = \infty \qquad l)\ \lim_{x\to-\infty} \dfrac{e^{2x}x^3}{e^x x^6} = 0$

$m)\ \lim_{x\to\infty} \dfrac{e^{x^2}x^3}{e^x x^6} = \infty \qquad n)\ \lim_{x\to-\infty} \dfrac{e^{x^2}x^3}{e^x x^6} = -\infty$

$o)\ \lim_{x\to\infty} \dfrac{(\ln|x|)^6}{x} = 0 \qquad p)\ \lim_{x\to-\infty} \dfrac{(\ln|x|)^6}{x} = 0$

$q)\ \lim_{x\to\infty} \dfrac{\sin^2 x}{x} - \dfrac{\cos^2 x}{x} = 0 \qquad r)\ \lim_{x\to-\infty} \dfrac{\sin^2 x}{x} - \dfrac{\cos^2 x}{x} = 0$

$s)\ \lim_{x\to\infty} \dfrac{\sqrt[3]{x^9+\sin^2 x}}{x^4+12} = 0 \qquad t)\ \lim_{x\to-\infty} \dfrac{\sqrt[3]{x^9+\sin^2 x}}{x^4+12} = 0$

Advanced Practice

Some more challenging problems:

$$\lim_{x\to\infty} \sqrt{x^2+2} - \sqrt{x^2-4} = 0$$

$$\lim_{x\to\infty} \sqrt{x^2+2} - \sqrt{x^3+2} = -\infty$$

$$\lim_{x\to\infty} \frac{(x+1)^{3x}\sqrt{x}}{\sqrt{x+1}x^{3x}} = e^3$$

$$\lim_{x\to 0} \frac{\sin 2x}{x^2\cos x} = \text{DNE}$$

$$\lim_{x\to\infty} \frac{\sin 2x}{x^2\cos x} = 0$$

$$\lim_{x\to 0} \frac{\tan x}{x^2+x} = 0$$

$$\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = 1$$

$$\lim_{x\to 3} \frac{x^3+6x-7}{x^4-81} = \pm\infty$$

$$\lim_{x\to 3} \left(\frac{x^2+4x-21}{x^2-9}\right)^{3/2} = \sqrt{\frac{125}{27}}$$

$$\lim_{x\to 0} \frac{\frac{1}{3+x} - \frac{1}{3-x}}{x} = \frac{-2}{9}$$

$$\lim_{x\to 3} \frac{1}{x-3}\left(\frac{1}{\sqrt{x+1}} - \frac{1}{2}\right) = \frac{1}{16}$$

$$\lim_{x\to -4} \frac{\frac{1}{\sqrt{13+x}} - \frac{1}{3}}{x+4} = \frac{-1}{54}$$

$$\lim_{x\to 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \frac{4}{3}$$

$$\lim_{x\to 4} \frac{x^{3/2}-8}{x-4} = -3$$