Problem Set 9: Limits

Key skills: Limit calculation

See Problem Set 11 for more advanced practice problems, with answers.

Practice Problems

$a)\ \lim\limits_{x \to -3} \left(\dfrac{\sin(\pi x)\cos(\pi x)}{x^2 - 4} + 2e^{\frac{x+3}{x}} \right) \left(x^2 - x + 1 \right)$

$b)\ \lim\limits_{x \to 4} \dfrac{1}{1 + \dfrac{1}{1 + \dfrac{1}{x}}}$

$c)\ \lim\limits_{x \to 0} \dfrac{\tan^2 x}{x} + x^2 \cot^2 x$

$d)\ \lim\limits_{x \to 0} \dfrac{x^3 - 2x^2 + x}{\tan x}$

$e)\ \lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

$f)\ \lim\limits_{x \to 0} \dfrac{4x - x^3}{3x + x^2}$

$g)\ \lim\limits_{x \to 0} \dfrac{(1 + x)^2 - 1}{x}$

$h)\ \lim\limits_{x \to 1} \dfrac{x^4 - 1}{x^2 + 2x + 3}$

$i)\ \lim\limits_{x \to 0^+} (1 + x)^{\frac{3}{x}}$

$j)\ \lim\limits_{x \to 0} \dfrac{\sin^2 x - \tan^2 x}{x^2}$

$k)\ \lim\limits_{x \to 0} \dfrac{\sin^2 x - \tan^2 x}{x^2}$

Answers

Remember to start by checking if the limit is indeterminate, e.g. $\frac{0}{0}$ or $\frac{\infty}{\infty}$ etc.

$$a)\ \lim_{x \to -3}\left(\frac{\sin(\pi x)\cos(\pi x)}{x^2 - 4} + 2e^{\frac{x+3}{x}}\right)\left(x^2 - x + 1\right) = 26 \qquad b)\ \lim_{x \to 4}\frac{1}{1 + \dfrac{1}{1 + \dfrac{1}{x}}} = \frac{5}{9}$$

$$c)\ \lim_{x \to 0}\frac{\tan^2 x}{x} + x^2 \cot^2 x = 2 \qquad d)\ \lim_{x \to 0}\frac{x^3 - 2x^2 + x}{\tan x} = 1$$

$$e)\ \lim_{x \to 2}\frac{x^2 - 4}{x - 2} = 4 \qquad f)\ \lim_{x \to 0}\frac{4x - x^3}{3x + x^2} = \frac{4}{3} \qquad g)\ \lim_{x \to 0}\frac{(1 + x)^2 - 1}{x} = 2$$

$$h)\ \lim_{x \to 1}\frac{x^4 - 1}{x^2 + 2x + 3} = 0 \qquad i)\ \lim_{x \to 0^+}(1 + x)^{\frac{3}{x}} = e^3 \qquad j)\ \lim_{x \to 0}\frac{\sin^2 x - \tan^2 x}{x^2} = 0$$

$$k)\ \lim_{x \to 0}\frac{\sin^2 x - \tan^2 x}{x^2} = 0$$

Questions $a)$, $b)$, $d)$, and $h)$ are solved by simple substitution. Questions $c)$, $j)$, $i)$, and $k)$ are solved by algebraic simplification to known limits. Questions $e) - g)$ are solved by algebraic manipulation and factoring.