Problem Set 2: Conics and Loci

Key skills: Analytic Geometry, Intersection

Practice Problems

Write the equations of the following lines and conics. Assume all conics (except the line) are centered at the origin, i.e. have equations in standard form. Check your answers by confirming that the given points solve the equations you find.

(a) The line through $(0, 1)$ with slope 3.
(b) The parabola through $(1, 7)$.
(c) The circle through $(2, -4)$.
(d) The ellipse through $(1, 1)$ and $(0, 6)$.

Find the intersection point(s) of the following pairs of lines and/or conics. Check your answers by confirming that the intersection point(s) satisfy the equations of both shapes.

(a) The line through $(-1, -2)$ and $(0, 0)$ and the parabola through $(-1, 4)$.
(b) The parabola through $(3, 3)$ and the hyperbola through $(2, 0)$ and $(4, 4)$.
(c) The line through $(-1, -1)$ with slope 2 and the hyperbola through $(2, 0)$ and $(-3, -1)$.
(d) The line through $(3, 1)$ with slope $-1/2$ and the ellipse centered and the origin which passes through $(-3, 0)$ and $(-2, 2)$.

Answers

Equations:

(a) $y = 3x + 1$
(b) $y = 7x^2$
(c) $x^2 + y^2 = 20$
(d) $\frac{35x^2}{36} + \frac{y^2}{36} = 1$

Intersections:

(a) The line has equation $y = 2x$ and the parabola has equation $y = 4x^2$. The intersection points are $(0, 0)$ and $\left(\frac{1}{2}, 1\right)$.
(b) The parabola has equation $y = \frac{x^2}{3}$ and the hyperbola has equation $\frac{x^2}{4} - \frac{3y^2}{16} = 1$. Note to find the intersection points here is quite tricky – you have a quartic and you need to use the quadratic formula with x^2 as the variable. Alternatively you can eliminate x^2 and use the quadratic formula with y as the variable. Either way, you find the use of the quadratic formula gives a negative square root; therefore, there are no intersection points.
(c) The line has equation $y = 2x + 1$ and the hyperbola has equation $\frac{x^2}{4} - \frac{5y^2}{4} = 1$. There are no intersection points.
(d) The line has equation $y = \frac{-x}{2} + \frac{5}{2}$ and the ellipse has equation $\frac{x^2}{9} + \frac{5y^2}{36} = 1$. The intersection points are $(\frac{-1}{3}, \frac{8}{3})$ and $(\frac{19}{7}, \frac{8}{7})$.