

Problem Set 1: Background Review

Key Skills: Algebra, Trigonometry, Fractions, Exponents, Logarithms.

Let x, y, a, b be real numbers, positive and non-zero when necessary. Determine whether the following statements are true or false.

$$\frac{a}{b} + \frac{a}{c} = \frac{a}{b+c} \quad T \quad F$$

$$(x - y)^2 = x^2 - y^2 \quad T \quad F$$

$$(2^a)^b = 2^{a+b} \quad T \quad F$$

$$\frac{1}{\frac{1}{x}} = \frac{1}{x} \quad T \quad F$$

$$x^{-2} = \frac{x}{2} \quad T \quad F$$

$$2^{3x} = 6^x \quad T \quad F$$

$$2^x 2^y = 2^{xy} \quad T \quad F$$

$$\frac{1}{\sin x} = \sec x \quad T \quad F$$

$$\frac{x+1}{x^2-1} = \frac{1}{x} - 1 \quad T \quad F$$

$$\frac{1}{x} + 1 = \frac{x}{x+1} \quad T \quad F$$

$$\sqrt{x^2 - x^4} = x^2 \sqrt{1 - x^2} \quad T \quad F$$

$$\frac{a}{c} - \frac{b}{d} = \frac{a-b}{c-d} \quad T \quad F$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ac}{bd} \quad T \quad F$$

$$x^3 - y^3 = (x - y)^3 \quad T \quad F$$

$$b \log_2 a = (\log_2 a)^b \quad T \quad F$$

$$\log_2 \frac{a}{b} = \frac{\log_2 a}{\log_2 b} \quad T \quad F$$

$$(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x + y \quad T \quad F$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{y} \quad T \quad F$$

$$\sin 2x = 2 \sin x \quad T \quad F$$

$$\sin\left(x + \frac{\pi}{2}\right) = \sin x + \sin \frac{\pi}{2} \quad T \quad F$$

$$\sin^2 x = 1 + \cos^2 x \quad T \quad F$$

$$\tan^2 x \cot^2 x = \sin^2 x \quad T \quad F$$

$$\frac{1}{1 + \frac{1}{x}} = \frac{x+1}{x} \quad T \quad F$$

$$\frac{1}{x^{-\frac{1}{2}}} = \frac{x}{2} \quad T \quad F$$

Answers

Let x, y, a, b be real numbers, positive and non-zero when necessary. Review the solutions to this true-false quiz. For each entry, construct a different right-side to the equation to fix the error.

$$\frac{a}{b} + \frac{a}{c} = \frac{a}{b+c} \quad T \quad F \quad \frac{ac+ab}{bc}$$

$$(x-y)^2 = x^2 - y^2 \quad T \quad F \quad x^2 - 2xy + y^2$$

$$(2^a)^b = 2^{a+b} \quad T \quad F \quad 2^{ab}$$

$$\frac{1}{\frac{1}{x}} = \frac{1}{x} \quad T \quad F \quad x$$

$$x^{-2} = \frac{x}{2} \quad T \quad F \quad \frac{1}{x^2}$$

$$2^{3x} = 6^x \quad T \quad F \quad 8^x$$

$$2^x 2^y = 2^{xy} \quad T \quad F \quad 2^{x+y}$$

$$\frac{1}{\sin x} = \sec x \quad T \quad F \quad \csc x$$

$$\frac{x+1}{x^2-1} = \frac{1}{x} - 1 \quad T \quad F \quad \frac{1}{x-1}$$

$$\frac{1}{x} + 1 = \frac{x}{x+1} \quad T \quad F \quad \frac{1+x}{x}$$

$$\sqrt{x^2 - x^4} = x^2 \sqrt{1 - x^2} \quad T \quad F \quad x \sqrt{1 - x^2}$$

$$\frac{a}{c} - \frac{b}{d} = \frac{a-b}{c-d} \quad T \quad F \quad \frac{ad-bc}{cd}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ac}{bd} \quad T \quad F \quad \frac{ad}{bc}$$

$$x^3 - y^3 = (x - y)^3 \quad T \quad F \quad x^3 - 3x^2y + 3xy^2 - y^3$$

$$b \log_2 a = (\log_2 a)^b \quad T \quad F \quad \log_2(a^b)$$

$$\log_2 \frac{a}{b} = \frac{\log_2 a}{\log_2 b} \quad T \quad F \quad \log_2 a - \log_2 b$$

$$(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x + y \quad T \quad F \quad x - y$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} \quad T \quad F \quad \frac{\sqrt{x}}{\sqrt{y}}$$

$$\sin 2x = 2 \sin x \quad T \quad F \quad 2 \sin x \cos x$$

$$\sin\left(x + \frac{\pi}{2}\right) = \sin x + \sin \frac{\pi}{2} \quad T \quad F \quad \cos x$$

$$\sin^2 x = 1 + \cos^2 x \quad T \quad F \quad 1 - \cos^2 x$$

$$\tan^2 x \cot^2 x = \sin^2 x \quad T \quad F \quad 1$$

$$\frac{1}{1 + \frac{1}{x}} = \frac{x+1}{x} \quad T \quad F \quad \frac{x}{x+1}$$

$$\frac{1}{x^{-\frac{1}{2}}} = \frac{x}{2} \quad T \quad F \quad \sqrt{x}$$