WHAT DO FLIPPED MATHEMATICS CLASSES ACCOMPLISH?
A LITERATURE REVIEW

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1. INTRODUCTION

At the past four meetings of the Alberta Mathematics Dialogue, we’ve heard pedagogy talks concerning
the flipped classroom model for instruction in undergraduate courses (mostly calculus courses). In addition
to one of these talks that I gave myself, I recall talks by Vincent Bouchard, Jeremy Sylvestre, Gerda de
Vries and Thomas Hillen. (Likely I have forgotten others, for which I apologize).

Sometimes, during the questions at the end of these talks, someone in the audience will ask for data that
shows whether this pedagogical method is effective. We’ve basically had to answer that we don’t have
much data other than the anecdotal accounts of our experience, the grade averages in our flipped classes
and the student feedback in our student evaluations. (The latter two pieces of evidence, of course, do not
form any kind of formal research evidence for many reasons, including the bias of the instructor and the
lack of ethical human research oversights.) We have been drawn to flipped classes because they seemed
like a good idea and, so far, our sense is that our flipped classes have been at least no worse than ordinary
lecture pedagogy.

Some audience members at these talks have found our answers unsatisfying, and with good reason. Last
year, in discussion after one of these talks, it was suggested that I should put together a literature review
to try to answer this question: what do flipped mathematics classes actually accomplish? This review is the
result of that suggestion. I will present this review at the 2019 Alberta Mathematics Dialogue (May 2-3,
University of Alberta Augustana Campus, Camrose, AB). I will also share the review with the organizers
of the First Year Math and Stats in Canada Conference (May 3-5, University of Alberta North Campus,
Edmonton, Alberta).

The main subject of this review is a collection of 51 mathematics education papers, all of which make
some kind of judgement about the success (or lack thereof) of flipped mathematics classes. Other than
two outliers in 2007 and 2010, all of these papers have been published between 2013 and 2019. I will
briefly summarize the math classes that the studies consider, the outcomes they seek to measure, and
their conclusions about the accomplishments of flipped classroom pedagogy.

I would like to point out, at the onset, that I am an instructor who uses flipped classrooms for calculus and
linear algebra courses and have done so for some years now. I’m not a neutral observer to this pedagogical
debate: I’m very fond of the flipped classroom model and would like to continue using it. That said, I’ve
tried to be neutral in the production of this review; in particular, I’ve not included or removed any studies
based on their negative or positive findings on flipped classroom pedagogy.
2. Scope and Goals of This Review

I'm concerned with evaluation of student outcomes of flipped mathematics classrooms. For this purpose, I've focused this review on studies of flipped classrooms which answer specifically make a judgement on student outcomes for mathematics courses. This naturally excludes many kinds of other literature.

This review excludes more general studies, including studies which may include mathematics among other subjects but for which mathematics is not mentioned in the title or abstract. There are many such general studies. To take just four examples that I noticed while searching for mathematics content, consider [AP19], [FGB14], [Hao16], and [Joh13].

This review excludes papers on flipped mathematics classrooms which focus on issues other than student outcomes. This means that I've excluded papers of the following topics.

- Design of flipped classes. (The books and reference materials of Bergman and Sams, particular [BS12] and [BS15], are very commonly cited as design materials.)
- Methods of implementation of flipped classes.
- Instructors' professional learning.
- Instructors' choice of pedagogical models and decisions about continuing with flipped models.
- Instructors' attitudes and experiences.
- Combinations of flipped classrooms with other novel pedagogies.
- Inquiries into the reasons or factors which determine success or failure of flipped classrooms.

Of these topics (all of which are valuable for their own purposes), I admit a particular interest in the last. This direction seems like the next natural step in the education research: once we have reasonable evidence of the success (or lack thereof) of flipped classrooms, it is natural to ask which factors influence that success. I found few studies of this type, but if I were to redo this review several years down the road, I would hope for progress in this direction. Here are four examples I did find: [LH16] and [HL17] for elementary school mathematics; [SK17] for secondary mathematics; and [CYH16] for post-secondary mathematics. This last study is particularly interesting for its focus on gender differences in motivation and performance.

Finally, several scholars have compiled similar meta-analyses and literature reviews, though I did not find any which directly paralleled my criteria and organization. Some examples of these are [AA18], [BV13], [LHC17], and [WLSS18]. Of these, [BV13] is very commonly cited as a relatively early source which defined the environment for research into flipped classrooms; many of the papers referenced in this review cite [BV13] for their definition of a flipped classroom and for a summary of the state of the research in 2013. Though Bishop and Verleger seem to adequately summarize the research at the time, basically none of the studies they cite are specific to mathematics classrooms. As I mentioned earlier, basically all of the papers I cite have been published since 2013. This research is very new.

Other than these exclusions, I made no other restrictions on the studies included. I have included all the studies I encountered which specifically address mathematics and make a statement about the outcomes of flipped classrooms as compared with more traditional delivery. I have not filtered by positive or negative results.
3. What Type of Mathematics Classroom is Flipped?

The studies investigate flipped pedagogy in a wide variety of mathematics classrooms. My experience and the majority of the recent discussion at the Alberta Mathematics Dialogue has been focused on first-year calculus courses. While calculus is the most common setting, there are many others.

In the 51 papers considered in this review, there are 17 which directly evaluate the use of flipped classrooms for calculus: [AD18], [Alb18], [AB15], [CBR18], [CFW17], [Jeo15], [MSS15], [MBX13], [Mac16], [Pet16], [SCZ15], [SMBX15], [SGE16], [WQNC17], [ZFGS15], [Zen17], and [ZT15]. In addition to calculus classrooms, [MSSS15] also considered an organizational behaviour class and [ZFGS15] also considered a finite mathematics class.

There are an additional 34 papers which evaluated the use of flipped classrooms in settings other than calculus classes.

- Nine papers are about mathematics in secondary schools: [BCC16], [Cla15], [EFT16], [Ful13], [LH17], [LLH18], [MG16], [RRCC17], and [TCT19].
- Nine papers are about college algebra or another generic first year university math course: [BRV14], [CCF18], [Cl17], [IC16], [Mat15], [Ogd15], [Ove14], [PMG18], and [SUF18].
- Five papers are about university statistics courses: [AO10], [Heu17], [MAZ16], [Str07] and [Wii13].
- Three papers are about university linear algebra courses: [HGS14], [MCS16] and [NKME17].
- Three papers are about university DEs courses: [BRB17], [LLY15], and [YLL15].
- Two papers are about mathematics content for elementary education students: [DD17a] and [DD17b].
- One paper is about a finite math course: [GBL15].
- One paper is about a numerical methods course: [Joh17]
- One paper is about a financial mathematics course: [LS18].

4. What Effects of Flipped Classroom are Evaluated?

Not all studies consider the same outcomes for flipped classrooms; a wide variety of achievements are considered. In addition to traditional performance on assignments, quizzes and exams, studies have measured failure rates, DFW rates, program retention, attendance, and future course performance. Studies have also tried to capture a wide range of students’ attitudes, perceptions and psychological risks. These outcomes include engagement, enthusiasm, anxiety, enjoyment, perception of learning, willingness to participate in future flipped classrooms, and perceived sense of value.

To make some sense of this breadth in the literature, I have chosen to oversimplify and group the outcomes into two broad categories: performance and attitudes. Performance captures achievement on assignments, quizzes and exams as well as measures like retention and future academic success. Attitudes is my catch-all for the long list in the previous paragraph; basically, anything relating to students’ perceptions, opinions, commitments and psychological concerns. Many studies address both performance and attitudes.

Of the 51 studies in this review, 40 considered performance outcomes and 44 considered attitudes. Of the 17 studies focused specifically on calculus courses, 15 considered performance and 15 considered attitudes.
5. Do Flipped Classes Accomplish Something?

The studies [Cla15] and [GBL+15] represent what I expected to find in this review: they reported no statistically significant changes in performance, but notable improvement in attitudes. This matches my personal experience, where I can’t really claim any performance improvement, but I receive (anecdotally) generally positive feedback from my students about the flipped classes. Moreover, I informally understood that this situation was the research consensus so far. I was incorrect in this assumption, and I was pleasantly surprised at the results of this review, which are summarized in this table.

<table>
<thead>
<tr>
<th>Group of Studies</th>
<th>Outcome Group</th>
<th>Positive</th>
<th>Mixed or Neutral</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Studies</td>
<td>Performance</td>
<td>28</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>All Studies</td>
<td>Attitudes</td>
<td>26</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>Calculus Studies</td>
<td>Performance</td>
<td>12</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Calculus Studies</td>
<td>Attitudes</td>
<td>10</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

I was surprised on two fronts. First, there are many more studies which are positive on performance than I expected. Second, the conclusions are more positive on performance than they are on attitudes, which was the reverse of my expectations.

6. Methodological Concerns

I am not a trained social scientist; I cannot claim expert judgement on the experimental design of the various studies cited in this review. However, even as a non-expert in this area, I can make some observations about the great differences among the studies.

- I noticed that the size of the studies varied considerably. Many studies considered two parallel sections of a course, often consisting of under 100 students. Several studies had 100 - 500 students. There were very few outliers with larger student populations, such as [Pet16] with 1471 students or [LS18] with 803 students. Some of the smaller studies included custom pre- and post-tests, custom surveys, interview and focus groups. Most of the larger studies produced their surplus of participants by simply using grades and existing student evaluation surveys as their data sources. Moreover, the size of the study may reflect the number of student who completed the course, but not the number who participated in surveys, interviews, or other more focused data collection.
- As already mentioned in the previous point, the style of data collection varied greatly as well. Some studies relied on grade averages and existing student survey, some used standard skill inventory exams, while other created custom data collection tools.
- The style of comparison with a control group also varied considerably. Most of the studies claimed some kind of comparison with non-flipped classroom (which is unsurprising, since I was specifically looking for conclusions that made this comparison). However, the nature of this comparison varied somewhat. Most commonly, studies compared a flipped class to a parallel section of the same topic. Many were careful to ensure common texts, assignments and exams. Some also had the same instructor teach both the flipped and control section. The most diligent studies had a careful description of a control group, adjusting their statistical analysis for various demographic differences that might exist between cohorts. In stark comparison, the most simplistic studies seemed to simply compare grades (or other outcomes) with previous terms’ averages.
As noted earlier in the report, I made a judgement call to include a wide variety of outcomes under the catch-all of ‘Attitudes’. I hope this grouping is a reasonable decision for the presentation of this review, but the variance under this heading should not be forgotten. In many of the ‘mixed’ results, the study may have found, for example, a positive effect in student engagement as well a negative effect in student anxiety. Whether any general trend can be taken from the attitudes results is questionable, considering the wide variety of attitudes considered.

7. Final Thoughts

Methodological concerns aside, as an instructor who enjoys the flipped classroom model, I take encouragement from these results. The rarity of negative results give me hope that, at the very least, flipped classroom pedagogy can stand alongside more traditional methods as a reasonable choice. This review indicates that the risks of significantly damaging students’ performance or enjoyment of courses is quite low. In addition, there is a reasonable chance that flipped classroom pedagogy has a real positive impact on performance and attitudes. This should be enough to encourage those of us who enjoy this pedagogy to keep improving and developing it. It should also help to make the case to our institutions that they should support the development and implementation of flipped classrooms.


Caleb Adams and Anthony Dove, Calculus students flipped out: the impact of flipped learning on calculus students’ achievement and perceptions of learning., Primus: Problems, Resources & Issues in Mathematics Undergraduate Studies 28 (2018), no. 6, 600.


Jonathan Bergmann and Aaron Sams, Flip your classroom: Reach every student in every class every day, International society for technology in education, 2012.


Jennifer Clinkenbeard, Flipping college algebra and pre-calculus: Impact on student outcomes.


This appendix lists the abstracts of 51 papers which specifically evaluated outcomes for flipped mathematics classrooms.

**Authors:** Anderson, Brennan [AB15]
**Title:** An experiment in flipped teaching in freshman calculus
**Abstract:** At Binghamton, Calculus 1 is taught to over 1000 students each fall in sections of about 30–40 students, with graduate student instructors teaching most sections. Despite having small classrooms instead of lecture halls, the satisfaction and performance of students has historically been poor. We had hoped to improve student success by changing how we teach and not by lowering our standards. In the fall of 2013, the Binghamton University Department of Mathematical Sciences undertook an experiment in flipped teaching with calculus 1 in which we compared a flipped model to our traditional lecture model. Overall, our quantitative analysis found moderate benefits to flipping over traditional methods for all groups studied. Informally, while student opinion varied, instructors largely were quite positive, finding that their students were more engaged and that instructors were able to give students more individualized attention.

**Authors:** Adams, Dove [AD18]
**Title:** Calculus students flipped out: the impact of flipped learning on calculus students achievement and perceptions of learning
**Abstract:** This assessment study examined one potential instructional method for improving student learning in Calculus for majors, flipped learning, in comparison with traditional lecture-based instruction. Results suggest that participation in flipped learning substantially improved growth in achievement over the semester, final exam grades, and final course grades. In contrast, there was no difference between the two groups in terms of students’ beliefs about learning mathematics. Additionally presented is a review of the changes made from the first year to the second year of implementing flipped learning, and how these changes are believed to have assisted with the increased impact on student learning.

**Author:** Albalawi [Alb18]
**Title:** The effect of using flipped classroom in teaching calculus on students’ achievements at University of Tabuk
**Abstract:** This study aimed at investigating the effectiveness of using flipped classrooms in teaching the Math2 course for the preparatory year’s students at the University of Tabuk, Saudi Arabia. The Math2 course was organized via an (ADDE) design model, with recorded videos of the topics included in the study; it was implemented by a Moodle platform and was available for the treatment group. Pre- and post-achievement tests were developed in calculus, being the topic of the Math2 course, and implemented for the two groups: the control group consisted of 45 students and the treatment group consisted of 47 students. The results revealed a significant difference in students’ performance, favoring the treatment group which was taught using the flipped classroom instruction. However, no statistical significance was found in students’ performance related to their majors (medicine, applied medical science, engineering, computer science, and science).
Author: Arano-Ocuaman [AO10]
Title: Differences in student knowledge and perception of learning experience among non-traditional students in blended and face-to-face classroom deliver

Abstract: The purpose of this study was to examine the efficacy of traditional and blended (partially online and partially face-to-face) course delivery methods. This study further examined the impact of using technology to improve student learning by providing meaningful learning in the areas of content delivery, communication and collaboration, evaluation and feedback, and personal learning experiences.

Non-traditional students enrolled in an elementary statistics course either delivered as a traditional course or a blended course participated in the study. It was hypothesized that students enrolled in the blended course would perform better and prefer this method of delivery compared to students enrolled in the traditional course. Student knowledge was assessed by test grades, course grades, and post-tests. Analysis of the first two indicators did not support the hypothesis that students in the blended course delivery would perform better than students enrolled in the traditional course delivery method. Contrary to the hypothesis, students in the face-to-face course scored higher in the post-test compared to the students in the blended course.

These contradictory results may suggest that the differences in teaching strategies and/or the use of technology have not resulted in a significant change or improvement in the performance of students. Past experience, familiarity with instructional format and types of assessment used may be considerations in the findings obtained.

Student perceptions were also measured. Results indicated that students in the blended course were more satisfied with using technology to facilitate and help them improve their learning than students in the traditional course.

Students in the blended course had more positives perceptions of their learning experiences than students in the traditional course in the following areas: (a) accessibility and availability of course materials; (b) use of web-based or electronic tools for communication and collaboration; (c) assessment and evaluation; and (d) student learning experiences with real-life applications. The perception of the majority of the students in both courses indicated a positive view of technology use in the classroom. The findings further suggest that student participants would choose blended course delivery as an alternative to face-to-face instruction.

Both course delivery methods emerged as enhancing the students appreciation of the integration of technology and recognizing the role of the teacher as the expert in the classroom, engaging students in meaningful learning. In spite of the emergence of technology in the classroom, the value of traditional instruction was indicated.

Authors: Bhagat, Chang, Chang [BCC16]
Title: The impact of the flipped classroom on mathematics concept learning in high school

Abstract: The present study aimed to examine the effectiveness of the flipped classroom learning environment on learner’s learning achievement and motivation, as well as to investigate the effects of flipped classrooms on learners with different achievement levels in learning mathematics concepts. The learning achievement and motivation were measured by the Mathematics Achievement Test (MAT) and Course Interest Survey (CIS), respectively. A pretest posttest quasi-experimental design was employed for this study. A total of 82 high-school students participated in this study, divided into experimental and control groups. The experimental group (41) was taught trigonometry using the flipped classroom method, while the control group (41) was taught by traditional teaching methods. The researchers employed independent sample t-test, analysis of covariance (ANCOVA), and multivariate analysis of variance (MANOVA) to analyze the data obtained. Findings indicated a significant difference in the learning achievement and motivation between the two groups,
with students using the flipped classroom performing better. Further analysis showed a significant difference in the performance of low achievers in the experimental and control groups.

**Authors:** Bego, Ralston, Barrow [BRB17]
**Title:** An intervention in engineering mathematics: flipping the differential equations classroom
**Abstract:** A flipped classroom design was implemented in the Differential Equations (DE) for Engineering course at the University of Louisville J.B. Speed School of Engineering. Student performance and completion rates were compared to a previous control semester of the course, and student satisfaction was measured. Overall, there were significantly fewer W’s (Withdrawals) in the course, meaning a significant improvement in course completion. Additionally, results showed significantly higher performance in non-First Time students, or students that were repeating the course. As DE is a required course for graduation, increasing student performance and decreasing W’s will improve graduation rates and is of great value to the university. Student satisfaction with the course format was slightly below average, however as this was the first implementation of the flipped design, it is expected that future semesters with slight modifications will be able to improve on this baseline.

**Authors:** Braun, Ritter, Vasco [BRV14]
**Title:** Inverted classroom by topic - a study in mathematics for electrical engineering students.
**Abstract:** The inverted classroom is a teaching model, where the students prepare for classroom by watching video lectures. The classroom time is then dedicated to individual practice. We evaluated a mathematics course for electrical engineering students throughout three semesters, where 20% of the topics were taught using the inverted classroom model. The aim was to find out whether the model can help to better address groups with large differences in prior knowledge in mathematics. We report mainly positive feedback from the students, although the opinions vary greatly between the groups. The students appreciate the increased amount of practice in the classroom as well as the possibility to learn at their own pace. Exam performance remained constant in the topics taught using the inverted classroom compared to previous semesters. The exam performance of weaker students also remained constant.

**Authors:** Combs, Bingham, Roper [CBR18]
**Title:** A model for inverting the advanced calculus classroom
**Abstract:** In this paper I discuss my experience in using the inverted classroom structure to teach a proof-based, upper level Advanced Calculus course. The structure of the inverted classroom model allows students to begin learning the new mathematics prior to the class meeting. By front-loading learning of new concepts, students can use valuable class time for exploring and solving complex problems, with the instructor available for guidance and assistance. This paper compares student performance in the inverted classroom with student performance in the direct instruction classroom. I also share student perceptions of the inverted classroom experience.

**Authors:** Carter, Carter, Foss [CCF18]
**Title:** The flipped classroom in a terminal college mathematics course for liberal arts students.
**Abstract:** The purpose of this study was to assess the effect of flipping the classroom on final exam scores in a terminal general education college mathematics course for a diverse student population. We employed a quasiexperimental design. Seven instructors collectively taught 13 sections of each pedagogy (flipped/traditional). Six hundred thirty-two students participated. Common final exams were graded concurrently. Mixed-model analyses were performed. Students in flipped sections scored 5.1 percentage points higher on average than those in traditional sections \( (p = .02) \) when
controlling for math SAT and financial aid status, an improvement of 7.8 points among Black students \((p < .01)\) and 1.0 points among Whites \((p = .67)\). The estimated average difference between White and Black students, conditional on covariates, was 5.2 percentage points in traditional sections \((p < .01)\) and –1.6 in flipped sections \((p = .39)\). The 6.8-point difference in achievement gap between pedagogies was statistically significant \((p < .01)\). Flipping the classroom was associated with improved student performance, particularly among Black students.

Authors: Cronhjort, Filipsson, Weurlander [CFW17]
Title: Improved engagement and learning in flipped-classroom calculus
Abstract: We report on an effort to measure the effect of replacing traditional lecture-based teaching in calculus with a flipped-classroom approach. We base the comparison between the two teaching models on data from three sources: (a) a Calculus Baseline Test, designed specifically for this purpose and given as pre-test and post-test; (b) a survey measuring student engagement; and (c) student achievement on the final exam. On the Calculus Baseline Test, we found that the normalized gain was 13% higher in the flipped-classroom group. Similarly, the flipped-classroom group scored significantly higher on the engagement survey. Also, the students of the flipped-classroom group performed much better than expected on the final exam of the course, with a substantial decrease in failure rate.

Author: Clark [Cla15]
Title: The effects of the flipped model of instruction on student engagement and performance in the secondary mathematics classroom.
Abstract: In many of the secondary classrooms across the country, students are passively engaged in the mathematics content, and academic performance can be described, at best, as mediocre. This research study sought to bring about improvements in student engagement and performance in the secondary mathematics classroom through the implementation of the flipped model of instruction and compared student interaction in the flipped classroom with a traditional format. The flipped model of instruction is a relatively new teaching strategy attempting to improve student engagement and performance by moving the lecture outside the classroom via technology and moving homework and exercises with concepts inside the classroom via learning activities. Changes in the student participants’ perceptions and attitudes were evidenced and evaluated through the completion of a pre- and post-survey, a teacher-created unit test, random interviews, and a focus group session. In addition, the researcher documented observations, experiences, thoughts, and insights regarding the intervention in a journal on a daily basis. Quantitative results and qualitative findings revealed the student participants responded favorably to the flipped model of instruction and experienced an increase in their engagement and communication when compared to the traditional classroom experience. The student participants also recognized improvements in the quality of instruction and use of class of time with the flipped model of instruction. In terms of academic performance, no significant changes were demonstrated between the flipped model of instruction students and those taught in a traditional classroom environment.

Authors: Dove, Dove [DD17a]
Title: Flipping preservice elementary teacher’s mathematics anxieties
Abstract: In preparing future elementary educators in mathematics, helping them overcome their anxieties of mathematics and teaching mathematics is paramount. This study examined how different instructional practices (in-class lecture, flipped learning with teacher-created videos, flipped classroom with Khan Academy videos) compared in improving students’ mathematics anxiety and
anxiety about teaching mathematics. Results suggest that, while all three methods improved students’ anxieties related to mathematics, flipped learning with teacher-created videos significantly had the greatest decreases in mathematics anxiety and anxiety about teaching mathematics. Survey responses and class interviews also suggested that flipped learning with teacher–created videos better aligned with course content and activities, thus helping students feel prepared and more confident before entering the classroom.

Authors: Dove, Dove [DD17b]
Title: How flipping much? Consecutive flipped mathematics courses and their influence on students’ anxieties and perceptions of learning.
Abstract: While studies have shown positive attributes related to flipped learning, especially in mathematics and statistics, there is limited understanding of how taking multiple flipped courses may impact students’ learning of mathematics and their perceptions of mathematics. Specifically, this study examined how completing consecutive flipped mathematics courses for elementary preservice teachers influenced their anxieties related to mathematics. Results suggest that students who took consecutive flipped mathematics courses significantly decreased in their mathematics anxiety compared to students taking their first flipped mathematics course. Survey responses suggest that there may be additional underlying issues that may also influence students’ perceptions, such as previous course instruction and students’ classroom expectations.

Authors: Esperanza, Fabian, Toto [EFT16]
Title: Flipped classroom model: effects on performance, attitudes and perceptions in high school algebra
Abstract: In this study, we evaluated student perceptions of the flipped classroom model and its effects to students’ performance and attitudes to mathematics. A randomized controlled trial with 91 high school algebra students was conducted. The experimental group participated in a year-long intervention of the flipped classroom model while the control group followed the traditional lesson delivery. Results of the year-end evaluation of this model showed positive student perceptions. An analysis of covariance of the algebra post-test score with learning model as treatment factor and pre-test as covariate resulted in a significant treatment effect at .05 level of significance. A paired-sample t-test by treatment group to compare pre-test and post-test math attitude scores resulted in a significant decrease in the control groups’ value of mathematics while the experimental group had a significant positive change in their confidence and enjoyment of mathematics.

Author: Fulton [Ful13]
Title: Byron’s flipped classrooms
Abstract: The article examines the use of a flipped classroom approach and digital education resources in the high school mathematics department of the Byron Independent School District 531 in Minnesota. Details are provided on how the mathematics teachers redesigned the curriculum and created their own lessons and materials through video lessons. Details are provided on academic achievement in the district following the change as well as the support from Superintendent Wendy Shannon.
Authors: Guerrero, Beal, Lamb, Sonderegger, Baumgartel [GBL+15]
Title: Flipping undergraduate finite mathematics: findings and implications.
Abstract: This paper reports on a research project that investigated the effects of a flipped instructional approach on student attitudes and achievement in a lower division university-level Finite Mathematics course. The project employed a mixed-methods design that included content exams, an attitude survey, open-ended student responses, observations, and instructor insights. Findings indicate that “flipping” allows instructors to repurpose class time for more student-centered interaction and problem solving, has positive effects on student attitudes toward mathematics, but has no significant impact on student learning over a more traditional approach. Discussion of implications focuses on factors that may influence the effectiveness of a flipped methodological approach.

Author: Heuett [Heu17]
Title: Flipping the math classroom for non-math majors to enrich their learning experience.
Abstract: Students’ learning experiences in an introductory statistics course for non-math majors are compared between two different instructional approaches under controlled conditions. Two sections of the course (n = 52) are taught using a flipped classroom approach and one section (n = 30) is taught using a traditional lecture approach. All sections are taught by the same instructor in the same semester. General perceptions as well as students’ understanding and retention of the course material are measured and compared. The flipped classroom students outperform their traditional lecture peers on exams, especially in terms of their mathematical problem-solving skills. The flipped classroom students are also more confident than their traditional lecture peers about their abilities and their understanding of the course material, crediting their understanding primarily to the in-class activities, which are made possible because the flipped classroom design promotes an experiential, active-learning environment without compromising content.

Authors: Ichinose, Clinkenbeard [IC16]
Title: Flipping college algebra: effects on student engagement and achievement
Abstract: This study compared student engagement and achievement levels between students enrolled in a traditional college algebra lecture course and students enrolled in a “flipped” course. Results showed that students in the flipped class had consistently higher levels of achievement throughout the course than did students in the traditional course, despite no differences in demographics. Moreover, students in the flipped course reported greater gains in affective variables related to mathematics than did students in traditional courses. In addition, this study found evidence that the flipped course experience was especially impactful for Hispanic women.

Author: Jeong [Jeo15]
Title: A case study of flipped learning in calculus of one variable on motivation and active learning
Abstract: Information Technology influenced on classroom to change the teaching and learning method. Recently, flipped learning method became a hot issue in education by using Information Technology. Learning management system that is introduced in our university in the spring semester 2015, made it possible to apply flipped learning method. So, we used the flipped learning method in a calculus course. In this paper, we found that flipped learning in Calculus we was a little bit affirmative in the aspect of motivation and active learning from students’ response on flipped learning method. We analyzed the reason that students were not so positive in continuing flipped learning even though they liked flipped learning a little bit better than traditional learning. We suggest what we pay attention to for applying the flipped learning method effectively.
Author: Johnston \[Joh17\]
Title: Implementing a flipped classroom approach in a university numerical methods mathematics course
Abstract: This paper describes and analyses the implementation of a ‘flipped classroom’ approach, in an undergraduate mathematics course on numerical methods. The approach replaced all the lecture contents by instructor-made videos and was implemented in the consecutive years 2014 and 2015. The sequential case study presented here begins with an examination of the attitudes of the 2014 cohort to the approach in general as well as analysing their use of the videos. Based on these responses, the instructor makes a number of changes (for example, the use of ‘cloze’ summary notes and the introduction of an extra, optional tutorial class) before repeating the ‘flipped classroom’ approach the following year. The attitudes to the approach and the video usage of the 2015 cohort are then compared with the 2014 cohort and further changes that could be implemented for the next cohort are suggested.

Authors: Lo, Hew \[LH17\]
Title: Using “first principles of instruction” to design secondary school mathematics flipped classroom: the findings of two exploratory studies.
Abstract: Flipping the classroom is a current pedagogical innovation in many schools and universities. Although interest in flipped classroom (or Inverted Classroom) continues to grow, its implementation so far has been driven more by teachers’ intuitive beliefs, rather than empirically-based principles. Many studies merely replace in-class instructions with videos and use class time for group discussions. But what instructional design framework should we use in planning the overall flipped classroom approach? This paper attempts to answer this question through two exploratory studies conducted in a Hong Kong secondary school. In Study 1, a flipped classroom Mathematics remedial approach was offered for underperforming students (\(n = 13\)) in Form 6 (Grade 12). In Study 2, high ability students (\(n = 24\)) in Form 6 participated in another flipped classroom Mathematics training approach. Both flipped classroom approaches utilized the First Principles of Instruction design theory. Paired t-test results indicated significant learning gains in both groups of students. Based on the suggestions of students and teacher as well as the existing literature, several recommendations for course planning, out-of-class learning, and in-class learning of flipped classroom are proposed.

Authors: Love, Hodge, Grangenett, Swift \[LHGS14\]
Title: Student learning and perceptions in a flipped linear algebra course
Abstract: The traditional lecture style of teaching has long been the norm in college science, technology, engineering, and mathematics (STEM) courses, but an innovative teaching model, facilitated by recent advances in technology, is gaining popularity across college campuses. This new model inverts or ‘flips’ the usual classroom paradigm, in that students learn initial course concepts outside of the classroom, while class time is reserved for more active problem-based learning and practice activities. While the flipped classroom model shows promise for improving STEM learning and increasing student interest in STEM fields, discussions to date of the model and its impact are more anecdotal than data driven – very little research has been undertaken to rigorously assess the potential effects on student learning that can result from the flipped classroom environment. This study involved 55 students in 2 sections of an applied linear algebra course, using the traditional lecture format in one section and the flipped classroom model in another. In the latter, students were expected to prepare for the class in some way, such as watching screencasts prepared by the instructor, or reading the textbook or the instructor’s notes. Student content understanding and course perceptions were examined. Content understanding was measured by the performance on
course exams, and students in the flipped classroom environment had a more significant increase between the sequential exams compared to the students in the traditional lecture section, while performing similarly in the final exam. Course perceptions were represented by an end-of-semester survey that indicated that the flipped classroom students were very positive about their experience in the course, and particularly appreciated the student collaboration and instructional video components.

**Authors:** Lo, Lie, Hew [LLH18]

**Title:** Applying “first principles of instruction” as a design theory of the flipped classrooms: findings from a collective study of four secondary school subjects

**Abstract:** The flipped classroom approach is a type of technology-enhanced pedagogy that has grown popular in education settings. An increasing number of empirical studies have evaluated this approach, but there is still no adequate theoretical framework for guiding the design and implementation of flipped classrooms. Furthermore, few such studies have been conducted in secondary school contexts, and the effects of flipped classrooms have not been adequately compared with those of non-flipped classrooms. This study aims to address these research gaps by applying the meta design theory “First Principles of Instruction” to design our flipped classroom approach. A two-stage study was conducted in two secondary schools, involving a total of 382 students and five teacher participants from four subject areas, namely mathematics, physics, Chinese language, and information and communication technology (ICT). Based on the experience of the pilot study (Study 1), we refined our flipped classroom model and examined its efficacy through a quasi-experimental design in the main study (Study 2). Although the students in the flipped ICT course had learning outcomes similar to those of students in the non-flipped ICT course, the levels of student achievement in the other three courses (i.e., mathematics, physics, and Chinese language) were improved after flipping, with a small to medium-sized effects. The design, benefits, and challenges of the model are discussed. We conclude by making several recommendations for practice, and suggesting ideas for further research.

**Authors:** Lape, Levy, Yong, Eddy, Handel [LLY15]

**Title:** Probing the flipped classroom: a controlled study of teaching and learning outcomes in undergraduate engineering and mathematics

**Abstract:** The article reports that a controlled study of teaching and learning outcomes in undergraduate engineering and mathematics. The inverted classroom model was implemented at Harvey Mudd College during the 2013-14 academic year in two courses: Engineering 82 and Math 45. In 2014, the National Science Foundation (NSF) provided support for this research endeavor, which will continue through the 2015-16 academic year.

**Authors:** Lopes, Soares [LS18]

**Title:** Perception and performance in a flipped financial mathematics classroom.

**Abstract:** The “flipped” classroom model is a new organizational design for the teaching and learning paradigm, as its name transmits, stands for the pedagogical switch of the traditional academic procedure as students’ first contact with the subjects is made outside the “four-wall classroom bounds”. Teachers’ role is transposed into a kind of guide and facilitator, indicating the way to go, avoiding to walk in a parallel path, or even ahead, but indicating the way to go, motivating students in their own knowledge construction, letting them lead the way, following and supporting, constantly and carefully monitoring their learning outcomes. Classroom time is consumed with open discussions, solving tasks and application problems, clarifying the supporting fundaments, in order to improve students’ engagement into their learning process in a collaborative environment.
A flipped model was implemented into a Financial Mathematics Course at ISCAP and the sample of our study consisted of 803 students, enrolled in 2014, 2015 and 2016. The main purpose of this paper is to investigate how the incorporation of the flipped classroom model into a Financial Mathematics class, affected students’ class training, learning, and achievement. The results obtained with this approach have shown a positive impact on students’ achievement overall.

Author: Maciejewski [Mac16]
Title: Flipping the calculus classroom: an evaluative study
Abstract: Classroom flipping is the practice of moving new content instruction out of class time, usually packaging it as online videos and reading assignments for students to cover on their own, and devoting in-class time to interactive engagement activities. Flipping has garnered a large amount of hype from the popular education media and has been adopted in a variety of contexts. Despite this high amount of interest, few studies have evaluated the effectiveness of classroom flipping on student academic outcomes. Specifically, no rigorous studies of the effects of flipping a mathematics course on students’ mathematical understandings and achievement appear in the literature. This article reports results from a control group study of flipping a large (N = 690), first-year university calculus course for life sciences students. Students in the flipped course sections on average outperformed their counterparts in the traditional sections on the final exam, though only by approximately 8%. A more detailed analysis reveals the true beneficiaries in a flipped classroom are those with high basic mathematical ability and low initial calculus knowledge. Gains for this group are considerable: approximately 10% on the final, with an effect size of $d = 0.56$, and comparable gains on an independent measure of calculus concept mastery. This study positions classroom flipping as an effective practice in undergraduate mathematics and calls for further research into the mechanisms behind its effectiveness.

Author: Mattis [Mat15]
Title: Flipped classroom versus traditional textbook instruction: assessing accuracy and mental effort at different levels of mathematical complexity
Abstract: Flipped classrooms are an instructional technology trend mostly incorporated in higher education settings, with growing prominence in high school and middle school (Tucker in Leveraging the power of technology to create student-centered classrooms. Corwin, Thousand Oaks, ). Flipped classrooms are meant to effectively combine traditional and online education by utilizing both in and out-of-class time. Despite positively reported implications of the flipped classroom instructional strategy, there is a deep shortage of literature and data that demonstrate advantages for student learning outcomes. The purpose of this preliminary study with directions for future investigations was to examine flipped classroom instruction versus a traditional classroom; specifically, an instructional video versus traditional textbook instruction to assess accuracy and mental effort at three levels of mathematical complexity. College-level nursing students who require mathematical mastery were used as a pilot test group in anticipation that this experience could be translated for larger data sets of variable age groups. Results indicated that accuracy increased and mental effort decreased with flipped instruction. Using Sweller’s cognitive load theory and Mayer’s cognitive theory of multimedia learning as theoretical frameworks, this study lends insight into designing effective instruction for learning environments that could benefit from a flipped classroom framework.
Authors: Mohamad, Alwani Zolkifli [MAZ16]
Title: The comparison of the effectiveness of the flipped classroom teaching method and interactive multimedia aided conventional teaching method for probability topic at Penang Matriculation College, Malaysia
Abstract: Probability is one of the topics in the syllabus of Mathematics semester II of Matriculation Program. Students often have difficulty in understanding the concept of probability during the teaching and learning process and they do not perform well on the quizzes. Therefore, this paper presents the flipped classroom teaching method and the interactive multimedia aided conventional teaching method in order to improve understanding of probability’s concept and also to provide alternative teaching methods to teachers. The study was conducted to evaluate and compare the effectiveness of both flipped classroom teaching method and the interactive multimedia aided conventional teaching method for probability topic. In addition, this study was also conducted to assess students’ perceptions of both teaching methods. The flipped classroom teaching method is using instructional videos while the conventional teaching methods is using interactive multimedia software as teaching aids. The selected research methods are experimental and descriptive survey. The samples comprised 261 Accounting students at the Penang Matriculation College. The effectiveness of the two teaching methods was assessed through an evaluation posttest after the teaching and learning process of probability topic completed. Independent t test results show that there is no significant difference at 0.05 significant levels between the flipped classroom teaching method and the interactive multimedia aided conventional teaching method in the population. The results of the given questionnaires show that the students exposed to the flipped classroom teaching method exhibit moderate perception while the students exposed the interactive multimedia aided conventional teaching method show a positive perception towards the use of interactive multimedia in teaching and learning. As a conclusion, this study contributes an insight and interesting findings towards the new learning environment in higher education in Malaysia.

Authors: McGivney-Burelle, Xue [MBX13]
Title: Flipping calculus
Abstract: In this paper we discuss flipping pedagogy and how it can transform the teaching and learning of calculus by applying pedagogical practices that are steeped in our understanding of how students learn most effectively. In particular, we describe the results of an exploratory study we conducted to examine the benefits and challenges of flipping a unit of study, the applications of the definite integral, in a Calculus II course. Data on student performance in flipped and non-flipped sections of the course are presented. In addition, students’ perceptions of the flipped unit are presented and discussed.

Authors: Murphy, Chang, Suaray [MCS16]
Title: Student performance and attitudes in a collaborative and flipped linear algebra course.
Abstract: Flipped learning is gaining traction in K-12 for enhancing students’ problem-solving skills at an early age; however, there is relatively little large-scale research showing its effectiveness in promoting better learning outcomes in higher education, especially in mathematics classes. In this study, we examined the data compiled from both quantitative and qualitative measures such as item scores on a common final and attitude survey results between a flipped and a traditional Introductory Linear Algebra class taught by two individual instructors at a state university in California in Fall 2013. Students in the flipped class were asked to watch short video lectures made by the instructor and complete a short online quiz prior to each class attendance. The class time was completely devoted to problem solving in group settings where students were prompted
to communicate their reasoning with proper mathematical terms and structured sentences verbally and in writing. Examination of the quality and depth of student responses from the common final exam showed that students in the flipped class produced more comprehensive and well-explained responses to the questions that required reasoning, creating examples, and more complex use of mathematical objects. Furthermore, students in the flipped class performed superiorly in the overall comprehension of the content with a 21% increase in the median final exam score. Overall, students felt more confident about their ability to learn mathematics independently, showed better retention of materials over time, and enjoyed the flipped experience.

Authors: Muir, Geiger [MG16]
Title: The affordances of using a flipped classroom approach in the teaching of mathematics: a case study of a grade 10 mathematics class
Abstract: Teaching secondary mathematics has a number of challenges, including the expectations that teachers cover the prescribed curriculum, help students learn difficult concepts, prepare students for future studies, and, increasingly, that they do so incorporating digital technologies. This study investigates a teacher’s, and his students’, perceptions of the benefits or otherwise of a flipped classroom approach in meeting these challenges, within a prescribed curriculum context. Data collection instruments included a survey designed to investigate the nature of students’ engagement with the flipped approach and semi-structured student and teacher interviews. Analysis of these data indicated that the teacher and students were positive about their experiences with a flipped classroom approach and that students were motivated to engage with the teacher-created online mathematics resources. The study adds to the limited research literature related to student and teacher perceptions of the affordances of the flipped classroom approach and has implications for secondary mathematics teachers who face the challenge of the twin demands of covering the prescribed curriculum and catering for a range of students’ learning needs.

Authors: McCallum, Schultz, Sellke, Spartz [MSSS15]
Title: An examination of the flipped classroom approach on college student academic involvement
Abstract: Colleges and universities remain attentive to developing and supporting ways to foster student academic success. These efforts have taken on more importance as student success, commonly measured by student learning achievement, has failed to meet expectations. For colleges and universities, the flipped classroom represents a student-centered method of fostering academic involvement that is recognized as a positive contributor to student success. This exploratory study examined the flipped classroom’s influence on student academic, student peer-to-peer and student-faculty involvement. The study involved 60 undergraduate students (28 male, 32 female) from three flipped classrooms consisting of courses in mathematics and business. Focus group interviews were conducted to gather student feedback regarding their behaviors and classroom engagement. Additionally, a brief survey was administered to collect demographic information as well as quantitative data regarding student perceptions. Findings indicated student academic involvement was present through note taking, viewing video lectures, active in-class learning and collaboration. Students cited peer-to-peer and student-faculty engagement as essential to relationship building, peer learning, and meaningful involvement with faculty.
Authors: Novak, Kensington-Miller, Evans [NKME17]
Title: Flip or flop? Students’ perspectives of a flipped lecture in mathematics.
Abstract: This paper describes students’ perspectives of a one-off flipped lecture in a large undergraduate mathematics service course. The focus was on calculating matrix determinants and was designed specifically to introduce debate and argumentation into a mathematics lecture. The intention was to promote a deeper learning and understanding through engagement with the added hope of instilling some passion for the subject. During the lecture, students were asked to vote with their feet, literally moving around the lecture theatre to form groups according to their shared favourite technique for calculating matrix determinants. Group discussions were then followed by a whole class debate facilitated by the lecturers, before they wrapped up the lecture by resolving the professional disagreements that had come to light during the debate. Following the lecture, data on student perspectives was gathered using both surveys and focus groups. Within this paper, we share the data and reveal the interesting results that emerged from our analysis. Despite remaining unconvinced as to whether flipped lectures are better for learning, students reported greater engagement and increased understanding of the material covered.

Authors: Ogden [Ogd15]
Title: Student perceptions of the flipped classroom in college algebra.
Abstract: The flipped classroom approach was implemented across three semesters of a College Algebra course. This paper is part of a larger design and development research study and focuses on student perceptions of the flipped classroom teaching approach. Qualitative methodology was used to describe how students perceived the instruction of their College Algebra course. Findings indicated that students felt that the teaching approach enabled them to ask more questions in class, the course components worked together to foster increased student learning, and the course design facilitated self-paced instruction.

Author: Overmyer [Ove14]
Title: The flipped classroom model for college algebra: effects on student achievement
Abstract: In the past few years there has been a substantial rise in the use and interest in a teaching and learning paradigm most commonly known as the flipped classroom. The flipped classroom model encompasses any use of using Internet technology to leverage the learning in a classroom, so that a teacher can spend more time interacting with students instead of lecturing. This is most commonly done by using teacher created videos that students view outside of class time. It is called the flipped class model because the whole classroom/homework paradigm is “flipped”. In its simplest terms, what used to be classwork (the lecture) is done at home via teacher-created videos and what used to be homework (assigned problems) is now done in class. Five sections of college algebra where taught using the flipped classroom model. Six sections of college algebra where taught using the traditional method of lecture and homework.

This quasi-experimental quantitative research compares sections of college algebra using the flipped classroom methods and the traditional lecture/homework structure and its effect on student achievement as measured through common assessments. In the traditional sections, students spent class time receiving lecture and reviewing homework and exams. Outside class time was spent on traditional homework. In the flipped sections, students viewed short video lectures and submitted basic homework solutions online outside of class time. Students then completed their homework assignments in class with the instructor. Some flipped section instructors also used collaborative group work, inquiry-based learning, and active whole-class discussions. All sections took common assessments for their final exam and completed a pre/post algebra readiness exam.
The exam data from the sections were analyzed and compared using regression and ANOVA methods with instructional method, gender, and ACT mathematics scores as independent variables. Final exam scores and pre/post algebra readiness exam scores were the dependent variables. The findings of this research show that there was not a statistically significant difference in the scores of students in the two groups, however students in the flipped sections did score slightly better than students in the traditional sections. Instructors of flipped sections who had previous classroom experience with inquiry-based and cooperative learning methods had sections with statistically significant higher common final exam scores. The results are followed by implications for teaching and recommendations for practice and further research.

Author: Petrillo [Pet16]
Title: On flipping first-semester calculus: a case study
Abstract: High failure rates in calculus have plagued students, teachers, and administrators for decades, while science, technology, engineering, and mathematics programmes continue to suffer from low enrollments and high attrition. In an effort to affect this reality, some educators are “flipping” (or inverting) their classrooms. By flipping, we mean administering course content outside of the classroom and replacing the traditional in-class lectures with discussion, practice, group work, and other elements of active learning. This paper presents the major results from a three-year study of a flipped, first-semester calculus course at a small, comprehensive, American university with a well-known engineering programme. The data we have collected help quantify the positive and substantial effects of our flipped calculus course on failure rates, scores on the common final exam, student opinion of calculus, teacher impact on measurable outcomes, and success in second-semester calculus. While flipping may not be suitable for every teacher, every student, and in every situation, this report provides some evidence that it may be a viable option for those seeking an alternative to the traditional lecture model.

Authors: Patterson, McBride, Gieger [PMG18]
Title: Flipped active learning in your mathematics classes without videos
Abstract: We explored the effectiveness of a flipped active learning pedagogy in a liberal arts mathematics course without video or interactive preparation. In both control and active learning classes, students were required to respond to a reading before class and take a quiz after class. During the active learning class, students worked together in groups on problems instead of listening to a lecture. This modest change in the in-class course structure showed that students were more willing to work with peers and ask questions of the instructor. Sources examined include common grade items as well as pre- and post-course attitude surveys.

Authors: Rogers, Rogers, Choins, Cox [RRCC17]
Title: Deliberate investigations of a flipped class.
Abstract: This article summarizes two action research projects completed under the direction of an eighth-grade mathematics teacher, who served as the mentor teacher for two teacher candidates from the University’s School of Education. This new professional development school was in its first two years of a one-to-one initiative. In AY15, the mentor teacher, intern, and university faculty collaborated on a study of their adoption of a flipped classroom approach to instruction. Two focus questions were examined: 1) What are eighth-grade mathematics students’ perceptions of a flipped classroom instructional approach? and 2) What impact does a flipped mathematics classroom have on eighth grade mathematics student homework submission? In AY16, the mentor teacher, a new intern, and the university faculty continued the examination of the flipped approach and added a third question; 3) What impact does a flipped mathematics classroom have on eighth
grade mathematics student engagement during class? Results from both years indicate a majority of student participants reported favorable responses to the flipped classroom approach. Homework submissions throughout the flipped segment of both studies remained high and student engagement in the flipped setting was higher than in the traditional setting. In the flipped setting sampled students spent more time working on mathematics topics and collaborating with peers than in the traditional setting; while sampled students in the traditional setting spent more time taking notes.

**Authors:** Sahin, Cavlazoglu, Zeytuncu [SCZ15]

**Title:** Flipping a college calculus course: a case study

**Abstract:** As online videos have become more easily available and more attractive to the new generation of students, and as new student-learning approaches tend to have more technology integration, the flipped classroom model has become very popular. The purpose of this study was to understand college students' views on flipped courses and investigate how the flipping affects their achievement in mathematics. We also studied how college students prepared for flipped classroom sections. Finally, college students' views were analyzed to see what they think about flipping in terms of benefits and preparation. Participants were 96 college students consisting of mostly freshmen & sophomores. We utilized descriptive statistics and paired t-test to analyze the data. Descriptive statistics revealed that participants preferred watching flip class videos (44%) over reading the sections from the textbook (17%) for preparation. Dependent t-test results showed that there is a statistically significant difference between students' average quiz scores from non-flipped sections and flipped sections. Students achieved significantly higher quiz scores in flipped sections than non-flipped ones. Overall, most of the students (83%) stated that flipped-taught lessons prepared them better.

**Authors:** Scott, Green, Etheridge [SGE16]

**Title:** A comparison between flipped and lecture-based instruction in the calculus classroom

**Abstract:** Purpose – The purpose of this paper is to enhance students’ mastery of course material in a second semester calculus course and assess the effectiveness of a traditional “lecture-based” instructional model compared to a modified “flipped classroom” model using a mixed method research design.

Design/methodology/approach – Evaluation of effectiveness was measured by having students in all four courses take the same pre-post exams and participate in a survey to assess their opinion of each instructional model. Undergraduate students covered the same content using two different modes of instruction.

Findings – Within a comparable group of students, participants in the flipped course had similar content knowledge gains as the lecture-based course. Based on responses obtained in blinded student surveys, the authors found that the use of an online homework system (WebAssign) and in-class quizzes were critical motivating factors that likely contributed to the increase in student performance. The flipped classroom instruction was more effective at increasing interest in the subject and promoting student discourse around problem solving. Taken together, the findings support the flipped classroom model as an equally effective means in which to disseminate key calculus concepts to undergraduate students.

Research limitations/implications – Findings from this study are limited to content knowledge gains in calculus over a 14-week time period, and student attitudes toward the mode of instruction they received. More research is needed to study the depth of learning that occurs in small group problem solving and interactions between faculty-and-student and student-to-student.

Originality/value – While many course redesigns focus on incorporating more project-based or laboratory-based learning opportunities, the redesign used in this study was focused on different
ways to deliver the same content. Results from this study suggest that a blended type of learning would be optimal for learning calculus concepts with a strategic use of video content concepts.

Authors: Schroeder, McGiveny-Burelle, Xue [SMBX15]
Title: To flip or not to flip? An exploratory study comparing student performance in calculus I
Abstract: The purpose of this exploratory, mixed-methods study was to compare student performance in flipped and non-flipped sections of Calculus I. The study also examined students’ perceptions of the flipping pedagogy. Students in the flipped courses reported spending, on average, an additional 1–2 hours per week outside of class on course content. Students enrolled in the flipped sections also performed better than students from the non-flipped sections on the common final exam. A follow-up examination of Calculus II grades revealed a statistically significant difference in course grades between these two groups of students.

Author: Strayer [Str07]
Title: The effects of the classroom flip on the learning environment: a comparison of learning activity in a traditional classroom and a flip classroom that used an intelligent tutoring system
Abstract: With the rise of technology use in college classrooms, many professors are open to structuring their classrooms in innovative ways. The classroom flip (or inverted classroom) is one such innovative classroom structure that moves the lecture outside the classroom via technology and moves homework and practice with concepts inside the classroom via learning activities.

This research compares the classroom flip and the traditional lecture/homework structure in two different college level introductory statistics classrooms. In the classroom flip classroom, an intelligent tutoring system (ITS) was used to deliver the lecture content outside the classroom. Students completed active learning projects in the classroom that often required the use of a spreadsheet computer program to help students work with the concepts in the course. In the lecture/homework classroom, students attended lectures on course content that included PowerPoint slides, and then students practiced with the course concepts by completing homework from their books outside of class.

The learning environment and the learning activity in both classrooms are investigated in this study with respect to activity theory and learning environments research. Students were given the College and University Classroom Environment Inventory (CUCEI) to measure both their learning environment preferences and their learning environment experiences. In addition, data were collected via field notes, classroom transcripts, student interviews, student focus groups, researcher journal entries, and student reflections. The quantitative data were analyzed using t-tests and MANOVA, and the qualitative data were analyzed using grounded theory methods.

The findings of this research show that classroom flip students were less satisfied with how the structure of the classroom oriented them to the learning tasks in the course. The variety of learning activities in the flipped classroom contributed to an unsettledness among students that traditional classroom students did not experience. Finally, the concept of student comfortability with learning activity is presented and developed in light of learning environments research.

Authors: Steen-Utheim, Foldnes [SUF18]
Title: A qualitative investigation of student engagement in a flipped classroom.
Abstract: The flipped classroom is gaining acceptance in higher education as an alternative to more traditional methods of teaching. In the current study, twelve students in a Norwegian higher education institution were in-depth interviewed about their learning experiences in a two-semester long mathematics course. The first semester was taught using flipped classroom and the second
semester using lectures, where both teaching modes contained a substantial amount of active learning. Overall, students report a more positive learning experience and higher engagement in the flipped classroom. The analysis revealed seven categories that the students highlight as especially conducive to their learning: commitment to peers, being recognized, feeling safe, instructor relationship, physical learning environment, learning with peers and using videos to learn new content. The results indicate that the affective dimension of student engagement is particularly prominent when students reflect upon learning in the flipped classroom.

Authors: Tse, Choi, Tang 

Title: Effects of video-based flipped class instruction on subject reading motivation

Abstract: Video-based flipped class instruction can strengthen the learning motivation of students. The effectiveness of flipped class instruction on teaching effectiveness and subject satisfaction has been evaluated previously. The present study aims to examine the impact of two aspects of subject reading motivation. A total of 100 secondary school students were recruited from 4 classes of 25 students (i.e., video-based flipped Mathematics class, traditional Mathematics class, video-based flipped Liberal Studies (LS) class and traditional LS class) in two local secondary schools. The same teachers taught both the traditional and video-based flipped classes in their subject. The students filled in questionnaires which measured: motivation for general reading; motivation for subject reading; academic subject satisfaction and perceived teaching effectiveness of the teachers. Analysis of covariance controlling for motivation for general reading revealed that students in the flipped classes reported significantly lower motivation for subject reading including reading curiosity, reading important and reading compliance (t (1, 98) = 10.52, p < 0.001; t (1, 98) = 7.68, p < 0.001; t (1, 98) = 20.39, p < 0.001, respectively). However, students in the flipped classes reported significantly higher satisfaction and teaching effectiveness than those in the traditional classes (t (1, 98) = −15.62, p < 0.001; t (1, 98) = −11.98, p < 0.001, respectively). A partial correlation controlling for motivation for general reading indicated that motivation for subject reading was negatively associated with academic subject satisfaction and perceived teaching effectiveness of teachers (rs ranged from −0.51 to −0.62, p < 0.001). Video-based flipped class instruction was not only related to increased academic subject satisfaction and teaching effectiveness but also associated with lower motivation for subject reading. These findings suggest that video-based flipped class instruction had limited capability to strengthen the learning motivation of students.

Author: Wilson

Title: The flipped class: a method to address the challenges of an undergraduate statistics course

Abstract: Undergraduate statistics courses are perceived as challenging by both students and instructors. Students’ attitudes, motivation, math anxiety, and preparedness can negatively impact the student and instructor experience and have the potential to negatively impact student learning. This article describes an attempt to address some of these challenges through structural and procedural changes to an undergraduate statistics course that is required for social science majors. The traditional lecture/homework structure of the course was “flipped” so that the majority of basic knowledge acquisition moved out of the classroom, making room for interactive activities during class time. The described changes had a positive impact on students’ attitudes toward the class and instructor as well as on students’ performance in the class.
Authors: Wasserman, Quint, Norris, Carr [WQNC17]

Title: Exploring flipped classroom instruction in calculus III

Abstract: In an undergraduate Calculus III class, we explore the effect of ‘flipping’ the instructional delivery of content on both student performance and student perceptions. Two instructors collaborated to determine daily lecture notes, assigned the same homework problems, and gave identical exams; however, compared to a more traditional instructional approach, the flipped instructor utilized videos to communicate more procedural course content to students out-of-class, with time in-class spent on more conceptual activities and homework problems. Findings from two semesters indicate similar performance on more procedural problems and small to moderate gains for the flipped students (\(N = 74\)) over their traditional counterparts (\(N = 77\)) on more conceptual exam problems. However, student perceptions remain mixed, with flipped students reporting increased communication during class but traditional students perceiving more effective use of class time, despite the gains in performance for flipped students.

Authors: Yong, Levy, Lape [YLL15]

Title: Why no difference? A controlled flipped classroom study for an introductory differential equations course.

Abstract: Flipped classrooms have the potential to improve student learning and metacognitive skills as a result of increased time for active learning and group work and student control over pacing, when compared with traditional lecture-based courses. We are currently running a 4-year controlled study to examine the impact of flipping an Introductory Differential Equations course at Harvey Mudd College. In particular, we compare flipped instruction with an interactive lecture with elements of active learning rather than a traditional lecture. The first two years of this study showed no differences in learning, metacognitive, or affective gains between the control and flipped sections. We believe that contextual factors, such as a strong group-work culture at Harvey Mudd College, contribute to the similar performance of both sections. Additionally, to maintain a rigorous experimental design, we maintained identical content across the control and flipped sections; relaxing this requirement in a non-study setting would allow us to take further advantage of educational opportunities afforded by flipping, and may therefore improve student learning.

Author: Zengin [Zen17]

Title: Investigated the use of the Khan Academy and mathematics software with a flipped classroom approach in mathematics teaching

Abstract: The purpose of this study was to determine the effect of the flipped classroom approach designed by using Khan Academy and free open source software on students’ academic achievement and to examine students’ views about this approach. The research was evaluated in the light of both qualitative and quantitative data. Twenty-eight students studying in the department of mathematics education in a state university in Turkey comprised the study group of the research which was conducted using a mixed methods research design. A double integral achievement test and an open-ended questionnaire about the flipped classroom approach were used as data collection tools. A Wilcoxon signed-rank test was used for the analysis of quantitative data and content analysis was used to analyse the qualitative data. According to the analysis of the research it was found that the flipped classroom approach designed with using both the Khan Academy and mathematics software increased student achievement in double integral. It was also found that this learning approach enhanced students’ understanding and provided visualization in mathematics teaching. Moreover, it was revealed that this approach promoted retention and made understanding much easier.
**Authors:** Zack, Fuselier, Graham-Squire, Lamb, O’Hara [ZFGS+15]

**Title:** Flipping freshman mathematics

**Abstract:** Our study compared a flipped class with a standard lecture class in four introductory courses: finite mathematics, precalculus, business calculus, and calculus 1. The flipped sections watched video lectures outside of class and spent time in class actively working on problems. The traditional sections had lectures in class and did homework outside of class. No statistical difference was found in test scores of the students, though qualitative data indicated potential problems with implementing the flipped pedagogy. Specifically, many students had negative opinions of the flipped model, and attitudes toward math in general tended to decline, comparatively, for students in the flipped class.

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**Authors:** Ziegelmeier, Topaz [ZT15]

**Title:** Flipped calculus: a study of student performance and perceptions.

**Abstract:** Flipping the classroom refers to moving lectures outside of the classroom to incorporate other activities into a class during its standard meeting time. This pedagogical modality has recently gained traction as a way to center the learning on students in mathematics classrooms. In an effort to better understand the efficacy of this approach, we implemented a controlled study at a small liberal arts college. We compared two sections of the entry-level course applied multivariable calculus I, with one section taught in a traditional lecture-based format and the other taught as a flipped classroom. During our study, we collected and analyzed data related to student performance, as well as perceptions of the approach and attitude toward mathematics in general. Students in both classes scored similarly on graded components of the course, and the majority of students were comfortable with the format of each section. However, some student perceptions and study habits differed.
This appendix gives a table summarizing the details of the 49 studies of flipped mathematics classes.

**Subject:** the particular mathematics class(es) used in the study.

**Size:** The number of students in the flipped classrooms involved in the study.

**Control:** The number of students in traditional lecture control groups. Zero indicates that the study was not compared to a control group.

**Style:** the method of evaluating the data, essentially qualitative or quantitative. The term ‘mixed’ includes both quantitative and qualitative methods. The term ‘informal’ indicates evaluations such anecdotal evidence, instructors’ impressions and conversations with students.

**Comparison:**

**Performance/Attitudes:** whether or not the study showed an effect, positive or negative, of the flipped instruction on the desired outcome. Neutral indicates no effect and mixed indicates a mixture of both positive and negative effects. The space is blank if the study didn’t address this outcome.

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